ADDITIONAL MATHEMATICS

Paper 0606/01

Paper 1

General comments

Attempts were of variable quality with some very good efforts but also some very poor ones. Some candidates appeared to lack the knowledge required in certain questions, others had the knowledge, even knew the processes, but were unsure of the application. Although most candidates seemed to have time to do all they could, there were those who ran short of time, often because of time spent on over-long efforts (especially in **Question 6** and in using binomial expansion in **Question 1**). Some candidates misquoted formulae (especially differentiation of quotients and products) and others produced answers without showing full working. In the latter case an incorrect answer cannot gain any available method marks. Standards of presentation remain variable.

Comments on specific questions

Question 1

Some candidates found this to be a hard opening question which involved having to realise that differentiation was needed and then having to cope with both composite function and product rule. The majority, however, coped well with the question, often losing the final mark only through algebraic and numerical slips rather than any error in differentiation. Some efforts using binomial expansion etc. were seen but very few were successful.

Answer: 17.

Question 2

Finding points *A* and *B* by removal of one variable and solution of the subsequent quadratic equation was done well by most candidates (although some found only x values or y values). From there on some found the equation of *AB* and stopped. Others found the gradient of the perpendicular but then found the equation of a line with this gradient but through either *A* or *B* instead of the mid-point of *AB*. A significant number of candidates thought the mid-point was found as half the difference of the x values (ditto the y values) rather than half the sum.

Answer : $4y = x + 41$.

Question 3

A large number of candidates made no attempt at all, or merely attempted to draw a triangle. Some candidates attempted to use the figures as given with no regard for the differing units. Fortunately most realised that they needed to work with consistent units – usually speeds, but some used distances and then brought in the 2 hours. Done correctly, with 45° the included angle between 150 and 500 (or 300 and 1000) in their triangle of velocities (displacements), cosine rule and then sine rule gave the required answers. Many candidates had 'the wrong triangle' with 135° as the included angle (by far the most common error), or had a triangle with 45° as the non-included angle and then used sine rule twice. Some candidates also found the required speed as the sum, or difference, of the two known speeds and still drew a triangle and attempted to use cosine or sine rule; others made the assumption of a right-angled triangle.

Some candidates who worked the question correctly spoilt their efforts by giving the angle required in part **(ii)** to the nearest whole number, rather than the 1 decimal place specified in the general rubric for angles in degrees.

A few candidates tackled the question using vector components with some success.

Answers: **(i)** 408 kmh^{-1}; **(ii)** 015.1°.

Question 4

Parts **(i)** and **(ii)** were well done by the vast majority of candidates, the errors that were seen resulting from mis-substitution of (0, 2) in part **(i)** and from using x = 0 as the x-axis in part **(ii)**. Part **(iii)** was less successfully tackled. Some candidates used the idea of reflection in the line $y = x$, others worked out algebraically an expression for the inverse and then plotted points. In either case only about half the attempts earned the two marks available.

Answers: **(i)** 3; **(ii)** (ln3, 0).

Question 5

Nearly all candidates made a reasonable attempt here. Common errors included in part **(i)** the belief that $\mathbf{B}^2$ is found by squaring each of the elements and in part **(ii)** the miscalculation of the determinant value as –10 (the latter reflecting the slips seen in manipulation of directed numbers throughout).

Answers: **(i)** $\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}$; **(ii)** $\begin{pmatrix} 3 & 1 \\ -5 & -1 \end{pmatrix}$.

Question 6

Few candidates obtained full marks on this question. Some didn't really make even a start while others missed 'short cuts' and became overwhelmed by long workings involving surds.

Two methods were prevalent for part **(i)**: multiplication of three brackets found from the given roots (usually, though not always, as (x – root)); solution of three equations found by substituting the three given roots into a general cubic equation with 1 as the coefficient of x^3. In the first method some candidates multiplied the factors relating to the 2nd and 3rd given root and multiplied the resultant by the 'first' bracket – giving the required answer fairly simply. Mainly, however, they multiplied the first two brackets together and then multiplied that result by the third bracket, and, as with the second method, produced long expressions involving surds.

A large number of candidates had work running, literally, into two pages just for part **(i)** – some eventually coming out to the correct answer, but many not.

Part **(ii)** was tackled well by most candidates – those not having an answer (or realising that their answer did not have integer coefficients) putting x = 3 into their original product of three brackets and using decimals.

Part **(iii)** was not well done in most cases, and the correct solutions seen usually came from creating a new cubic equation replacing x by $-x$ and solving. Relatively few realised that the new cubic would have roots which were the negative of the original roots. Some candidates took their cubic and changed all signs (i.e. $-f(x)$ not $f(-x)$) and many were hampered by a wrong answer in part **(i)** which created great problems in attempting to solve.

Answers: **(i)** $x^3 - 6x - 4$; **(ii)** 5; **(iii)** $x = 2$ or $-1 - \sqrt{3}$ or $-1 + \sqrt{3}$.

Question 7

On the whole this was a question on which good marks were earned by the candidates. A few candidates did not realise that calculus was needed, and others took differentiation to result in displacement and integration in acceleration but these were not too common. The only real problem for those using the correct links was the constant of integration with some candidates not using the initial conditions and obtaining two equations in three unknowns.

Although well done on the whole there were some fairly frequent errors: in differentiation leaving qt as qt or 4 as 4, and in the solution of equations dealing incorrectly with the fractional term in the displacement.

Answers : p = 1.5, q = 5.

Question 8

Most candidates quoted and/or used the quotient rule correctly (although some misquoted – with the terms in the numerator being switched or linked by a + sign). Some candidates used the product rule correctly with their second function being (cos x) but some were clearly confused and used the product rule for a quotient.

Candidates should be aware that, when asked to 'show that' a result is true, every step of their working/thinking should be absolutely clear to examiners e.g. '$\frac{1+\sin x}{1-\sin x} = \frac{1}{1-\sin x}$' is missing a step and will not earn full marks, nor will cancelling if it is not clear whether an entire bracket e.g. (1 + sin x) is being cancelled or whether just the sin x terms.

In part **(ii)**, in spite of the instruction to use the result given in part **(i)**, a large number of candidates started afresh attempting to integrate the expression given for the curve – to no avail. Of those using the result given in part **(i)**, a surprisingly large number did not know how to deal with the '2'. Some ignored it, others divided by it, others did multiply by it but only multiplied one of the terms in the numerator. The use of limits was generally correct but the (incorrect) use of degrees occurred in a significant proportion of scripts.

Answer: **(ii)** 4.

Question 9

There were three types of response to part **(a)**. Some candidates gained full marks having both the knowledge required (laws of logarithms, change of base etc.) and the confidence to perform the necessary manipulation. Others had (some of) the knowledge but limited skill or confidence in the topic while others seemed not at all familiar with logarithms. The candidates displaying the necessary knowledge and skills were in the minority.

Part **(b)** was even less rewarding for candidates, with even a large proportion of those gaining full marks in part **(a)** floundering here. The main problem amongst those who had at least some idea was that they thought $(\log_3 y)^2$ was equal to $2\log_3 y$, resulting in many cases in the statement that $4\log_3 y = 8$. Of those who produced a three term quadratic some thought the solution – 4 gave no answer, while others went into decimals and gave 0.012, i.e. 3 decimal places rather than the required 3 significant figures.

Answers: **(a)(i)** 4^u, **(ii)** $2 - u$, **(iii)** $\frac{3}{2u}$; **(b)** $y = 9$ or $\frac{1}{81}$.

Question 10

The most common error in part **(i)** was to say $\cos x = \frac{1}{12}$, but of those who reached $\cos 4x = \frac{1}{3}$ and found an angle which was then divided by 4 an alarming number stopped at one solution. They seemed to think that $4x$ rather than x had a maximum value of 180°.

There was some confusion amongst a number of candidates about amplitude and period, some giving one of them as the answer for the other. Period was also often thought of as number of cycles.

Maximum and minimum values were often given in part **(iv)** incorrectly though they were shown correctly on a sketch graph. The sketch itself was often perfectly correct but common errors included x going only to 90°, not having a number of cycles to match their period, and end points not suggesting maxima.

Answers: **(i)** 17.6° or 72.4° or 107.6° or 162.4°; **(ii)** 3; **(iii)** 90°; **(iv)** maximum 2, minimum –4.

Question 11 EITHER

Amongst Centres where this was the predominant choice there were some good efforts. Most such candidates managed to draw the correct straight line and to make a good attempt at a and b, either by re-arranging their equation and using m and c, or by using two points on their line substituted into the appropriate equation and solving simultaneous equations. These candidates also tried part **(iii)** by again re-arranging the equation to make x the subject.

Among the weaker of such candidates the algebra often let the candidates down, and for those using simultaneous equations an *xy* value from the graph was often wrongly used as an *x* value in the original equation.

Amongst the candidates from Centres where there was no preference for either alternative it was often weaker candidates who chose this alternative – and in many cases part **(i)** was all that was attempted, and some had no idea of what was involved and merely drew a (curve) graph of *y* against *x*.

Answers: **(ii)** a = 7.1 to 7.3, b = 0.7 to 0.9; **(iii)** gradient = value of a, intercept = value of $-b$.

Question 11 OR

A large number of candidates scored full marks on this question and several more lost just the last mark by taking (and using) $\sqrt{117}$ as 10.8 and getting 58.3 (to 3 significant figures), rather than 58.5 (to 3 significant figures).

The finding of the coordinates of *C* and *D* was usually clear and straightforward but the one common error was the assumption of an isosceles triangle in order to find the coordinates of *D* and then use of the coordinates of *D* to show that triangle *BAD* is isosceles. Such a circular argument gained candidates little credit.

Finding the area in part **(ii)** was achieved by a number of means, often by the 'array method' even though candidates had the lengths of two sides at right angles to each other.

Answers: **(i)** *C*(15, 15), *D*(15, 3); **(ii)** 58.5.

ADDITIONAL MATHEMATICS

Paper 0606/02

Paper 2

General comments

This paper proved to be rather more challenging than the corresponding paper of last year. Areas in which candidates were particularly weak were the vectors of **Question 3**, especially the concept of a unit vector, and the arrangements and selections of **Question 7**. In the context of the latter question many candidates were not prepared to give a brief indication of their reasoning; this was also the case in **Question 11(i)**.

Comments on specific questions

Question 1

The product rule and $\frac{\mathrm{d}}{\mathrm{d}x}(\ln x)$ were generally well-known and used correctly so that few candidates had any difficulty in finding $\frac{\mathrm{d}y}{\mathrm{d}x}$. Some candidates went no further than $\frac{\mathrm{d}y}{\mathrm{d}x}$ and some confused the elements of the chain rule, obtaining $\frac{2}{3}$ or $\frac{3}{2}$ rather than 6. Many candidates used the increments δy and δx instead of the rates of change, $\frac{\mathrm{d}y}{\mathrm{d}t}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}$.

Answer: 6 units per second.

Question 2

Many candidates obtained full marks and it was pleasing to see how often statements such as '$(5 \times 3) \times (3 \times 1) \Rightarrow (5 \times 1)$' occurred. Common errors were carrying out five separate matrix multiplications or using matrices which were not conformable for multiplication. The answer was occasionally given as a row when a column was appropriate, or vice versa, and some candidates summed the elements to give an answer of 54. Others generated incorrect elements by including the 'games played' column in their calculations.

Answer: (18 13 12 7 4) or its transpose, as appropriate.

Question 3

(i) Only the better candidates were able to evaluate p correctly. The expression $(0.28)^2 + p^2$ was frequently seen but few candidates knew its value and were thus unable to equate it to 1.

(ii) The vector $\overrightarrow{AB}$ was frequently correct and for many candidates this was the only worthwhile part of the question. The vector $\overrightarrow{AB}$ was commonly equated to $0.28\mathbf{i} + p\mathbf{j}$ so that q was given as 11.72. Relatively few candidates used the fact that the components of $\overrightarrow{AB}$ and of the unit vector were in the same ratio and most of the successful solutions arose from first finding the magnitude of $\overrightarrow{AB}$ as 25. Some candidates realised that $\frac{12-q}{\sqrt{(12-q)^2+24^2}} = 0.28$ and a few successfully used this to find q, although for most the manipulation required by this approach proved to be too difficult.

Answers: **(i)** 0.96; **(ii)** 5.

Question 4

(a) Most candidates understood that the derivative of $e^{\tan x}$ involved $e^{\tan x}$ and there were few instances of $e^{\tan x - 1}$. Some incorrect answers, which were seen quite frequently, were sec $e^{\tan x}$ and tan $e^{\tan x}$. Poorly expressed solutions, e.g. '$e^{\tan x} = \sec^2 x\ e^{\tan x}$ ', were quite common.

(b) The integration of e^{1-2x} usually led to an integral containing e^{1-2x} which was multiplied, correctly by $-\frac{1}{2}$ on many occasions, sometimes incorrectly by –2 or some function of *x*. Evaluation of the definite integral was almost always correct in principle with only a few candidates unthinkingly dismissing the value at the lower limit as zero.

Answers: **(a)** $e^{\tan x}\sec^2 x$; **(b)** 0.859.

Question 5

(i) A number of weaker candidates thought the area of the triangle was given by $AB \times AC$. The method of rationalising the denominator was understood by the vast majority of candidates; most multiplied their fraction above and below by $\sqrt{3}-\sqrt{5}$, resulting in – 2 in the denominator. This was a fairly frequent cause of error with candidates omitting the minus sign at some stage or dealing with it incorrectly. Some candidates were unable to resolve $\sqrt{75}$ and $\sqrt{45}$ into $5\sqrt{3}$ and $3\sqrt{5}$ respectively. Occasionally a candidate offered a neat solution by expanding $\frac{1}{2}\left(\sqrt{3}+\sqrt{5}\right)\left(a\sqrt{3}+b\sqrt{5}\right)$, equating to $1+\sqrt{15}$ and solving the resulting simultaneous equations in *a* and *b*.

(ii) Candidates who were able to handle the surds in part **(i)** correctly almost invariably found *c* and *d* correctly. Few candidates were unaware of the method involved.

Answers: **(i)** $4\sqrt{3}-2\sqrt{5}$; **(ii)** $76-14\sqrt{15}$.

Question 6

(a) Many candidates scored both marks. There were fewer errors in part **(ii)** than in part **(i)**, where $(P\cup C)\cap B'$, rather than $(P\cap C)\cap B'$, sometimes occurred.

(b) Better candidates had little difficulty in dealing with this part with many explaining how they had arrived at their value. Some candidates simply gave one answer to each part, usually '$\text{n}(F\cap S)=6$, $\text{n}(F\cup S)=20$', without comment as to whether each of these values was a maximum or a minimum, whilst others gave the same number for both the maximum and the minimum value e.g. 'Maximum = 20, minimum = 20'.

Answers: **(b)(i)** Maximum 10, minimum 6; **(ii)** Maximum 20, minimum 16.

Question 7

This was without doubt the most difficult question on the paper. Vary many candidates scored little or nothing and only a handful obtained full marks.

(a) In many cases there was no explanation, merely a string of numbers, factorials, ${}^nC_r$ s or ${}^nP_r$ s. Some candidates drew diagrams indicating the possible positions of Andrew and Brian so that it was possible to understand how a factor of 3 arose, but frequently the interchange of Andrew and Brian was overlooked. Other candidates regarded (Andrew ... Brian) as one element so that, having found the number of arrangements of this element, subsequent multiplication by 3! could account for the arrangement of this element with the 2 remaining boys; unfortunately, in many cases the number of arrangements of (Andrew ... Brian) was taken to be 3! or 3! × 2, whereas the 3! should have been ${}^5P_3$ or 3! × ${}^5C_3$.

(b) Only extremely rarely did a candidate show any understanding of this part. The one feature common to many of the answers was the appearance of ${}^6C_3$ either as the answer itself or in combination with various other factors.

Answers: **(a)** 720; **(b)** 50.

Question 8

(i) The most common approach was to evaluate r by taking the exponent of x in ${}^8C_r\, x^{8-r}\left(\frac{k}{x^3}\right)$ to be zero. Having found $r = 2$ a frequent error was to take the term independent of x to be $28k$ rather than $28k^2$, leading to $k = 9$. Of those who listed the terms of the expansion, the weaker candidates were unable to select and then simplify the relevant term. Others made errors with indices such as $(x^3)^2 = x^5$.

(ii) This part of the question was rather poorly done. Many of the candidates who correctly obtained $k = 3$ did no more than find $24x^4$, the second term in the expansion of $\left(x+\frac{3}{x^3}\right)^8$. It was somewhat surprising that so many failed to realise that it was necessary to combine two terms, particularly as it was given that 252 was one of the terms in the expansion of $\left(x+\frac{k}{x^3}\right)^8$.

Answers: **(i)** 3; **(ii)** –39.

Question 9

Some of the weakest candidates were unable to express the surface area in terms of x and h, but most candidates had little difficulty with parts **(i)** and **(ii)**. In part **(iii)** many who found $\frac{dV}{dx} = 40 - 4x^2$ and $x = \sqrt{10}$, all of which was perfectly correct, then crossed this work out, preferring to substitute $h = \frac{4x}{3}$ in $2x^2h$ so that $\frac{dV}{dx} = 0$ then led to $x = 0$. Few candidates seemed to have a clear idea of how to proceed to the required result.

Answer: **(i)** $\frac{60-2x^2}{3x}$.

Question 10

(a) Squaring and adding terms led to $2\sec^2x + 2\mathrm{cosec}^2x$ at which point many candidates stopped; some candidates simply, and without justification, recorded the sum of the six terms as $2\sec^2x\ \mathrm{cosec}^2x$. Those who replaced $\sec x$ and $\mathrm{cosec}\, x$ by $\frac{1}{\cos x}$ and $\frac{1}{\sin x}$ respectively could usually proceed from $2\sec^2x + 2\mathrm{cosec}^2x$ to $2\sec^2x\ \mathrm{cosec}^2x$, but those using $\sec^2x = 1 + \tan^2x$ and $\mathrm{cosec}^2x = 1 + \cot^2x$ were rarely successful.

(b) Cot y was usually replaced by $\frac{\cos y}{\sin y}$ or, all too frequently, by $\frac{\cos}{\sin}y$, but rather fewer candidates replaced the ensuing $\sin^2y$ by $1 - \cos^2y$. A number of candidates did not recognise the equation thus obtained as a quadratic in $\cos y$ and could not proceed. Others tried to factorise the quadratic or to apply the invalid reasoning that $\cos y\,(3\cos y + 2) = 3$ implies $\cos y = 3$ or $3\cos y + 2 = 3$. Some of those who found the values 0.721 and – 1.387 took these to be values of y rather than $\cos y$.

Answer: **(b)** 0.77 and 5.52.

Question 11

(i) The weakest candidates did not attempt this part. Of those that did attempt it almost all reached the given answer, although not all such attempts were valid. Many candidates failed to give reasons for the steps taken while others introduced a letter, usually x, to denote an angle, without ever stating which angle x referred to. Many solutions were only approximate, candidates using ΔAOC to show that angle AOC was 1.86 radians and that $5 - \pi$ was also 1.86. Some of the candidates using decimal approximations in this part continued to use these in the later parts of the question and as a consequence were insufficiently accurate.

(ii) Some candidates found the shaded area via {sector $OACB$ – (ΔACO + sector COB)} but the more direct method of (sector OAC – ΔOAC) was the usual approach. Some of the weakest candidates used 2.5 rather than $5 - \pi$, but most knew how to find the area of a sector. Finding the area of ΔOAC sometimes caused problems, with candidates having to use $\frac{1}{2}AC \times$ (perpendicular from O to AC) rather than $\frac{1}{2} \times 12^2 \times \sin(5 - \pi)$. A few candidates quoted, and used successfully, the formula for the area of a segment, $\frac{1}{2}r^2(\theta - \sin\theta)$.

(iii) The length of the arc AC was usually found correctly. The length of the straight line AC caused fewer problems than the area of ΔAOC in part **(ii)**, and was frequently evaluated via the cosine rule. A few candidates found the perimeter of sector OAC rather than the perimeter of the shaded region.

Answers: **(ii)** 64.8 cm^2; **(iii)** 41.5 cm.

Question 12 EITHER

(I) Candidates attempting to complete the square were frequently unable to avoid errors in arithmetic, The most successful method, used by only a few candidates, involved expanding $a(x + b)^2 + c$ and equating coefficients.

(ii) Very few candidates used the values of b and c from part **(i)** to give the coordinates of the stationary point as $(-b, c)$, but many candidates easily found the correct coordinates by using the calculus. What was disappointing was that so few candidates saw any connection between parts **(i)** and **(ii)** and so were quite content to offer incorrect values of b and c in part **(i)** and correct coordinates in part **(ii)**.

(iii) Some candidates realised that $f^2(0)$ could quickly be evaluated via f {f (0)} = f (3) but many felt it was necessary to first obtain $f^2(x)$. Some of the weakest candidates understood $f^2(x)$ to be $\{f(x)\}^2$ but a much more common error was to omit the power 2 so that $f^2(x)$ became $2(2x^2 - 8x + 3) - 8(2x^2 - 8x + 3) + 3$. While some candidates substituted $x = 0$ in their formal expression for $f^2(x)$, thus obtaining $2(3)^2 - 8(3) + 3$, others continued with $f^2(x)$, trying to obtain a polynomial in x; this latter strategy usually ended in failure with $(2x^2 - 8x + 3)^2$ often taken to be $4x^4 - 64x + 9$.

(iv) Only the better candidates understood what was required here.

(v) Those candidates answering part **(i)** correctly usually understood how to approach this part of the question but barely a handful obtained full marks. The ambiguity of sign obtained from taking the square root was either not considered or was dismissed, so that the answer given was $2 + \sqrt{\frac{x+5}{2}}$.

Answers: **(i)** $2(x - 2)^2 - 5$; **(ii)** $(2, -5)$; **(iii)** -3; **(iv)** 2; **(v)** $g^{-1}(x) = 2 - \sqrt{\frac{x+5}{2}}$.

Question 12 OR

This proved to be rather more popular than the preceding alternative.

(i) Most candidates found the critical values 1 and 5, but a considerable number of these candidates failed to obtain the correct set of values of x. The most successful strategy was to solve $10 - x^2 + 6x = 15$ and then decide whether the required values lay between or outside the critical values. Candidates working with the inequality and beginning with $10 - x^2 + 6x > 15$ sometimes failed to change the direction of this inequality when changing the signs of the terms. Occasionally weaker candidates assumed that $x^2 - 6x - 10$ was the same as $-x^2 + 6x + 10$ and so began with $x^2 - 6x - 10 > 15$.

(ii) Although candidates usually understood what was required, many failed to deal correctly with the signs involved. The terms $-x^2 + 6x$ appeared to be regarded as $-(x^2 + 6x)$ so that a common answer involved $-(x + 3)^2$; in such cases a frequently took the value 1, from $10 - (x + 3)^2 - 9$.

(iii) As in the preceding alternative, very few candidates realised there was any connection between this part of the question and part **(ii)**, but there were many correct solutions using the calculus. Some candidates using $\frac{dy}{dx} = 0$ to find $x = 3$ did not proceed to find the corresponding value of y.

(iv) The vast majority of candidates obtained gf(x) correctly; those who failed to do so usually offered what was actually fg(x). Most understood that the discriminant was involved, although some were unable to identify the elements a, b and c of the discriminant correctly and others used $b^2 - 4ac > 0$ rather than $b^2 - 4ac = 0$.

Answers: **(i)** $1 < x < 5$; **(ii)** $19 - (x - 3)^2$; **(iii)** (3, 19); **(iv)** 38.