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ADDITIONAL MATHEMATICS

Paper 0606/01

Paper 1

General comments

There was a considerable improvement in the work of the candidates in this second year of the examination. Some of the 'newer' topics, particularly matrices and relative velocity, are still causing problems for some candidates. However, there were many very good scripts and the standard of algebra and arithmetic was good. Presentation of work was generally acceptable, though there are still Centres whose candidates divide a page into two halves. This makes marking very difficult and is of no advantage to the candidates.

Comments on specific questions

Question 1

The vast majority of candidates realised the need to use the discriminant of a quadratic equation in either x or in y . Only about half of these correctly used $b^2 - 4ac < 0$; the rest used $b^2 - 4ac = 0$ or > 0 or < 0 . Candidates obtaining the quadratic equation in y were more successful since algebraic and arithmetic errors were common in formulating the equation in x , especially in the expansion of $\left(\frac{k-x}{3}\right)^2$, which was all too often replaced by $\frac{(k-x)^2}{3}$. Manipulation of the linear inequality in k also caused problems with many candidates unable to move numbers or variables from one side of the inequality to the other or to realise the effect of multiplying through by a negative quantity.

Answer: $k < -6$.

Question 2

The majority of candidates attempted to express each side of the given equation as powers of x and many attempts realised that $8^{-x} = 2^{-3x}$ and that $4^{\frac{1}{2}x} = 2^x$. Unfortunately, a large proportion thought that $\frac{2^a}{2^b} = \frac{2^c}{2^d}$ implies $\frac{a}{b} = \frac{c}{d}$ rather than $a - b = c - d$. There were also a few solutions (some of them successful) in which logarithms were used.

Answer: $x = 1.6$.

Question 3

This was very well answered with the majority of candidates realising that if the expression was written as $f(x)$ then $f(3) = 0$ and $f(-2) = 15$. The solution of the simultaneous equations presented few problems. It was very pleasing that the error of taking either $f(-3) = 0$ or $f(2) = 15$ was rarely seen.

Answer: $a = 1, b = -11$.

Question 4

There were many very good solutions. Most candidates realised the need to square $\sqrt{3} - \sqrt{2}$ and, apart from a few weaker candidates who obtained such results as '3 - 2' or '3 + 2' or '1 - 2 $\sqrt{6}$ ', the majority proceeded to obtain the height of the cuboid as $\frac{4\sqrt{2} - 3\sqrt{3}}{5 - 2\sqrt{6}}$ m. Although most candidates realised the need to multiply top and bottom by $5 + 2\sqrt{6}$, it was obvious that some candidates had never met this technique. Those who applied the technique were generally correct with their value for a and for b .

Answer: $2\sqrt{2} + \sqrt{3}$.

Question 5

This was poorly answered. Most candidates realised the need to integrate but attempts were poor. Taking $\int 6 \sin\left(3x + \frac{\pi}{4}\right) dx$ as either $\pm 6 \cos\left(3x + \frac{\pi}{4}\right)$ or as $\pm 18 \cos\left(3x + \frac{\pi}{4}\right)$ or as $+2 \cos\left(3x + \frac{\pi}{4}\right)$ were all common errors. The use of limits was generally correct, but finding the correct upper limit defeated most candidates. Most realised the need to set y to 0, but failed to solve $6 \sin\left(3x + \frac{\pi}{4}\right) = 0$ to give $x = \frac{\pi}{4}$.

Answer: 3.41.

Question 6

There were only a handful of correct solutions, even from very good candidates. Only a small percentage realised that using vectors to achieve the actual velocity of the plane as $330\mathbf{i} - 110\mathbf{j}$ enabled the bearing ($90^\circ + \tan^{-1}\left(\frac{1}{3}\right)$) and the time ($273 \div \sqrt{(110^2 + 330^2)}$) to be calculated directly. Many candidates ignored the question completely, a significant number took the actual velocity of the plane as $(280\mathbf{i} - 40\mathbf{j}) - (50\mathbf{i} - 70\mathbf{j})$ whilst a significant minority ignored vector notation and used trigonometry to find the angles and resultants associated with each of the wind and plane's velocity in still air, before using both the cosine and sine rule to evaluate the two answers. One had to admire the staying power of the few that were completely correct.

Answers: (i) 108.4° ; (ii) 47 minutes.

Question 7

There was a significant improvement in the attempts at the matrix question. Most candidates were able to write down three appropriate matrices to represent the data of the question, but these were often non-compatible for multiplication. Although many correct answers were seen – such statements as:

$\begin{pmatrix} 40 \\ 50 \\ 50 \\ 60 \end{pmatrix} \begin{pmatrix} 7.3 & 5.9 & 5.2 & 4.4 \end{pmatrix} = 1111$ were very common. Candidates must realise that, in a question of this type,

AB \neq **BA** and that a correct order must be shown.

Answer: \$1111.

Question 8

This was well answered and there were many correct solutions. Differentiation of $\frac{\ln x}{2x+3}$ presented a few difficulties with candidates often incorrectly quoting the quotient rule or failing to realise that $\frac{d}{dx}(\ln x) = \frac{1}{x}$. It was very common to see $\frac{d}{dx}(\ln x)$ expressed as '1' or as 'lnx' or as 'e^x'. Simplification of $\frac{(2x+3) \times \frac{1}{x} - 2 \ln x}{(2x+3)^2}$ as $\frac{2x+3-2 \ln x}{x(2x+3)^2}$ was another common error. Parts (ii) and (iii) were well answered, though in part (ii) it was common for candidates to recognise that $\delta y = \frac{dy}{dx} \times p$, but to then fail to substitute $x = 1$. Part (iii) was also generally correct with only a small proportion of candidates failing to recognise the need to divide $\frac{dy}{dt}$ by $\frac{dy}{dx}$.

Answers: (i) $\frac{2x+3-2 \ln x}{x(2x+3)^2}$; (ii) 0.2p; (iii) 0.6 units per second.

Question 9

- (a) Many attempts were completely correct – others were affected by the use of incorrect equations such as 'sec²x = tan²x – 1' or 'sec²x = 1 – tan²x' or even 'secx = 1 + tanx'. Other errors included solving 4sec²x + 8secx = 5 as 'secx = 5' or '4secx + 8 = 5'. Most however obtained cosx as –0.2 or +2 and realised that the latter was impossible. Only a few solutions were seen in which candidates failed to realise that secx is the reciprocal of cosx. Finding the angles in the correct quadrants was well done.
- (b) This was poorly answered, mainly due to a failure to cope with radians. Many attempts evaluated tan⁻¹3.2 as 72.6° and then assumed 2y + 1 = 72.6. A few candidates worked throughout in radians and converted at the end – unfortunately the '+1' was still taken as 1°. Many candidates failed to realise the need to add multiples of π to 1.27 to obtain further solutions. A few weaker candidates expanded tan(2y + 1) as tan2y + tan1.

Answers: (a) 113.6°, 246.4°; (b) 3.28 radians.

Question 10

Only a small minority realised that f(x) < 5 in part (i); most made no attempt at it. Many attempts at part (ii) were correct; most of the others attempted to take logarithms of both sides before isolating the e ^{$\frac{1}{2}x$} . Such attempts as '3e ^{$\frac{1}{2}x$} = 5' leading to ' $\frac{1}{2}x \ln 3 = \ln 5$ ' were seen on many scripts. The sketch in part (iii) was poorly done with the curve in the first quadrant usually shown with a correct y-intercept, but often shown either as a straight line or curving in the wrong direction. Part (iv) was very well done, though as in part (ii) a common error was to take logarithms before isolating the e ^{$\frac{1}{2}x$} .

Answers: (i) f(x) < 5; (ii) 1.02; (iv) $2 \ln \left(\frac{5-x}{3} \right)$.

Question 11

Most candidates coped comfortably with this question, usually by solving three pairs of simultaneous equations, though a few solutions were seen in which the coordinates of B were deduced directly by considering the vector $\overrightarrow{OB} (= \overrightarrow{OC} + \overrightarrow{CB})$. Occasional errors occurred when D was incorrectly taken as the mid-point of OC or when AD was assumed to be perpendicular to OA rather than AB or OC . The solution of the appropriate simultaneous equations was excellent. Part (ii) was also well answered though occasionally candidates misread $OABC$ and used $DABC$, or used 'area' instead of 'perimeter'.

Answers: (i) $C(6, 3)$, $B(8, 9)$, $D(4, 2)$; (ii) 26.1.

Question 12 EITHER

This was the more commonly answered of the two alternatives. Only a minority of candidates, however, were able to correctly answer part (i), though most realised the need to obtain an expression for x in terms of r from the perimeter. Common errors were to use $2\pi r$ instead of πr or to include additional lengths.

Attempts at the area varied considerably, though ' $\frac{1}{2}\pi r^2 + 2rx$ ' was usually correct. Most candidates realised

the need to use Pythagoras' Theorem to find the height of the triangle as $\frac{3}{4}r$, though common errors were

either to take $\sqrt{\left(\frac{5r}{4}\right)^2 - r^2}$ as $\frac{r}{4}$ or to assume that the triangle was right-angled and take the area as

$\frac{1}{2} \times \left(\frac{5r}{4}\right)^2$. Most candidates realised the need to differentiate and to set $\frac{dA}{dr}$ to zero, though $\frac{d^2A}{dr^2} = 0$ was

also seen. The differentiation was generally correct, but the solution of $\frac{dA}{dr} = 0$ caused some difficulty with r occurring twice in the equation.

Answers: (i) $x = \frac{1}{2}\left(125 - \pi r - \frac{5r}{2}\right)$; (ii) 18.8.

Question 12 OR

There were only a small number of attempts at this question. Most of the solutions seen realised the need to use similar triangles or trigonometry to obtain h in terms of r . Unfortunately many of these assumed $\frac{h}{r} = \frac{30}{12}$ instead of $\frac{h}{12-r} = \frac{30}{12}$. In part (ii), the differentiation and solution of $\frac{dV}{dr} = 0$ was generally accurate and candidates were able to obtain correct values for the volume of both the cylinder and the cone.

Answers: (i) $h = \frac{5(12-r)}{2}$; (ii) 2010 cm³.

<p>Paper 0606/02</p>

<p>Paper 2</p>

General comments

The overall performance of candidates appeared to be somewhat better than in last year's examination. Areas of particular difficulty were the manipulation of logarithms required in **Question 3** and, to a lesser extent, in **Question 11**, integration of the exponential function of **Question 6** and the mathematical comprehension needed to deal with **Question 8**, especially part (iii).

Comments on specific questions**Question 1**

This question proved to be an easy starter to the paper with the majority of candidates earning full marks. Most candidates took the simpler initial step of eliminating x rather than y but occasionally weaker candidates substituting $\frac{x+11}{4}$ for y failed to square properly, obtaining $x^2 + 121$ as the numerator or leaving the denominator as 4. Some obtained the correct x and y values for the points A and B but then interchanged the order of the values when writing down the coordinate pairs. A few used $\left(\frac{x_1-x_2}{2}, \frac{y_1-y_2}{2}\right)$ as the mid-point and some thought they had completed the question on finding the coordinates of A and B .

Answer: (5, 4).

Question 2

Most candidates found little difficulty with this question. Weaker candidates were often unable to bring the two fractions to a common denominator and simplify. Some could not proceed further after reaching a common denominator of $(1 - \sin\theta)(1 + \sin\theta)$ whilst others took this to be $(1 - \sin\theta)^2$. Occasionally the fractions were broken down e.g. $\frac{1}{1 - \sin\theta} \equiv \frac{1}{1} - \frac{1}{\sin\theta}$ while in some cases $1 - \sin\theta$ was replaced by $\cos\theta$.

Answer: 2.

Question 3

Many candidates failed to answer this correctly usually through poor handling of logarithms rather than difficulty with the change of base. Thus $\log_4(x - 4)$ became $\log_4 x - \log_4 4$ or $\frac{\log_4 x}{\log_4 4}$ and hence $\log_4 x$.

Those who correctly took $\log_4(x - 4)$ to be $\frac{\log_2(x-4)}{\log_2 4}$ frequently went on to $\log_2\left(\frac{x-4}{4}\right)$ or $\log_2(x - 4) - \log_2 4$. Some took $\log_4(x - 4)$ to be $\log_2(x - 4)^2$ or took $\log_2 x$ to be $\frac{1}{2} \log_4 x$. Only a few confused the change of base rule e.g. $\log_2 x = \frac{\log_4 2}{\log_4 x}$.

Answer: 8.

Question 4

- (i) This was almost always correctly answered although a few candidates simply shaded B' .
- (ii) This proved to be the most difficult part of the question with many answers involving $B' \cup C'$.
- (iii) Candidates frequently overlooked the significance of brackets so although $B \cup (A \cap C)$ is correct, $B \cup A \cap C$ is ambiguous; on the other hand $A \cap C \cup B$ is not. Although quite correct, some candidates gave unnecessarily complicated answers using $(A \cap B)$ or $(B \cap C')$ for B .

Answers: Various, including (ii) $A \cap B' \cap C'$ or $A \cap (B \cup C)'$; (iii) $B \cup (A \cap C)$ or $(B \cup C) \cap A$.

Question 5

There were many completely correct answers. Some of the weakest candidates only felt comfortable dealing with $(\pm)(1 - 3x)^5$; the first three terms were then left in ascending powers of x or rearranged to be in descending powers of x . Apart from such candidates errors were usually sign errors, failure to use binomial coefficients properly or taking powers of $3x$ to be $3x^5$, $3x^4$, etc. rather than $243x^5$, $81x^4$, etc. Most candidates understood how to proceed in part (ii), although weaker candidates sometimes considered only one product. There were also quite a number of transcription errors e.g. 405 in part (i) becoming 450 in part (ii).

Answers: (i) $243x^5 - 405x^4 + 270x^3$; (ii) 135.

Question 6

The weakest candidates made very little headway with this question or omitted it altogether. Of those attempting the question most understood that ‘instantaneous rest’ implies $v = 0$ and were able to obtain $t \approx 2.30$; only a few attempted to solve $\frac{dv}{dt} = 0$ or $s = 0$. Again, most understood that integration was necessary but many were unable to perform this correctly; common errors included $\frac{e^{-t+1}}{-t+1}$, $\frac{e^{-t}}{-t}$, and even $\frac{(e^{-t} - 0.1)^2}{2}$, as well as $-40e^{-t} - 0.1t$ and $-40(e^{-t} - 0.1t)$. Most attempts were based on finding an expression for s using the indefinite integral and introducing c , a constant of integration; unfortunately, this was frequently followed by the dismissal of c with the comments ‘when $t = 0$, $s = 0 \therefore c = 0$ ’. Attempts using the definite integral and evaluation of $\int_0^{2.30}$ were often successful although, here again, evaluation at the lower limit was sometimes ignored.

Answer: 26.8m.

Question 7

Most candidates knew how to find the inverse of a 2×2 matrix. Some weaker candidates confused the operations as to which elements to interchange and which to alter sign, whilst others omitted division by the determinant or used the modulus of the determinant. Although there was a considerable number of arithmetical mistakes in parts (i) and (ii), candidates generally knew how to combine matrices under subtraction or addition. Common errors in part (i) were using \mathbf{B}^{-1} instead of \mathbf{B} or taking \mathbf{C} to be $2\mathbf{A}^{-1} - \mathbf{B}$. Candidates were much less familiar with multiplication of matrices and, in part (ii), \mathbf{D} was frequently attempted via \mathbf{AB}^{-1} rather than $\mathbf{B}^{-1}\mathbf{A}$. Less able candidates attempted to divide \mathbf{A} by \mathbf{B} , using ratios of corresponding elements, or to obtain the product of \mathbf{A} and \mathbf{B}^{-1} , in whichever order, by multiplying corresponding elements. Occasionally elements of a product were obtained using difference rather than sum e.g. $a_1b_1 - a_2b_3$. Quite a number of candidates ignored the word ‘Hence’, which was intended to imply the use of \mathbf{B}^{-1} in part (ii), and proceeded from $\mathbf{D} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ via the solution of equations.

Answers: $-\frac{1}{2} \begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix}$, $\frac{1}{8} \begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix}$; (i) $\begin{pmatrix} 3 & -1 \\ -5 & 7 \end{pmatrix}$; (ii) $\frac{1}{8} \begin{pmatrix} 9 & 5 \\ 14 & 6 \end{pmatrix}$.

Question 8

Parts (i) and (ii) were usually attempted and were frequently correct but a considerable number of candidates used ${}^n\text{P}_r$ instead of ${}^n\text{C}_r$. In part (ii) most realised that the solutions consisted of choosing 6 out of 7, but some tackled the question by considering 5 red and 1 yellow, and 4 red and 2 yellow; in the latter case candidates not only confused P and C but also used multiplication where addition was appropriate and vice versa. Other attempts at part (ii) included ${}^{10}\text{C}_5 \times {}^{10}\text{C}_2$, ${}^6\text{C}_5 \times {}^6\text{C}_2$ and $210 - {}^{10}\text{C}_3$. Part (iii) of this question proved to be a stumbling block for the vast majority of candidates and correct solutions were infrequent. It was very rare that a candidate appreciated that part (ii) could be a help towards part (iii) and that the only further piece of information required was the number of selections containing no yellow rose i.e. ${}^8\text{C}_6$. Some candidates tackled the question by considering separately the selections in which only two colours were present i.e. 3 pink and 3 red, 2 pink and 4 red, etc. ... and then subtracting the sum from 210. The usual correct approach was to consider the selection containing at least one of each colour i.e. 1 pink and 3 red and 2 yellow, etc.; unfortunately most attempting this method were unable to analyse the situation completely, omitting at least one of the six possible cases. A common answer was 1050, obtained by selecting one rose of each colour, $3 \times 5 \times 2$, and multiplying by ${}^7\text{C}_3$.

Answers: (i) 210; (ii) 7; (iii) 175.

Question 9

- (i) All but the very weakest candidates recognised the function as a product and differentiated as such. The only recurring error, the omission of the factor 4 from the derivative of $(4x - 3)^{\frac{1}{2}}$, was only seen occasionally. Weak candidates were frequently unable to combine correctly the terms obtained by differentiation and the numerator sometimes contained a constant term as well as a multiple of x .
- (ii) Candidates were divided into two groups: (a) those who saw no connection with part (i) and almost invariably produced complete nonsense, the exceptions being the one or two candidates with the knowledge to offer a reasonable attempt using substitution or parts, (b) those who realised that the integration involved the reverse of the process in part (i), in which case the only errors were due to an incorrect value of k , usually 4, or a misuse of the value of k i.e. multiplying by 12 rather than dividing.

Answers: (i) 12; (ii) $6\frac{2}{3}$.

Question 10

Part (i) was usually answered correctly although some candidates, both here and in other parts of the question, appear to be unable to work in radians, having to calculate angles and degrees and then convert. In parts (ii) and (iii) some candidates employed a circular argument, using the angle of 0.485 radians given in part (iii) to calculate the length of DE and then using this length to show that angle $DOE \approx 0.485$ radians. Others, quite legitimately found angle DOE in part (ii) by applying the sine rule to triangle OCD in order to find angle CDO , hence angle DOE and so DE from triangle DOE . Others quickly found the length of DE by making CO the hypotenuse of a right-angled triangle and calculating $8\sin 1.2$. But it was surprising to see that so many candidates unnecessarily apply the sine rule to right-angled triangles, an approach which in part (ii) leads to $\frac{8 \sin 1.2}{\sin 90^\circ}$; this did not always lead to 7.46 in that $\sin(90 \text{ radians})$ was sometimes used. The

length of OE was frequently taken to be 16 cm which led, in part (iii) to $\tan DOE = \frac{7.46}{16}$ rather than $\sin DOE$, whilst others used Pythagoras in order to fallaciously calculate the length of OD from $(16^2 + 7.46^2)^{\frac{1}{2}}$; in part (iv) $OE = 16$ cm was used to obtain the area of triangle ODE as $\frac{1}{2} \times 16^2 \times \sin 0.485$.

Quite frequently candidates correctly, but rather laboriously, calculated the area of sector DOB by subtracting the area of sector AOD from that of sector AOB . Others went to considerable trouble to find the area of trapezium $OCDE$ in order to combine with the area of ACD , which they took to be a sector of a circle, centre C . A few considered DEB to be a triangle and others regarded DEB as a segment, evaluating $\frac{1}{2}r^2(\theta - \sin \theta)$ with $r = 16$ and $\theta = 0.485$. A rather pleasing solution was offered by candidates employing symmetry and the segment formula with $\theta = 0.970$, dividing the result by 2.

Answers: (i) 1.2; (ii) 7.46 cm; (iv) $9.10 - 9.35\text{cm}^2$.

Question 11

Some of the weakest candidates omitted this question or merely plotted R against v . A relatively small number plotted $\lg v$ against $\lg R$ or used logarithms to base e rather than base 10. Scales were frequently ill-chosen and many candidates spoiled the activity of their graph by rounding all values to two significant figures when the scales chosen allowed plotting to three significant figures. Many candidates began quite correctly with $\lg R = \lg k + \beta \lg v$ but there were also cases of $\lg R = \beta \lg k + \beta \lg v$. Candidates usually knew that the gradient and the intercept on the $\lg R$ axis were relevant; the gradient was usually correct but the intercept was far too often incorrect in that one or both of the axes were broken and consequently did not pass through the origin. Despite expressing $\lg R$ correctly, the intercept was occasionally taken to be k rather than $\lg k$. Some candidates preferred to solve simultaneous equations, either in R and v or in $\lg R$ and $\lg v$, in order to find β and k , but these attempts were generally less successful than those which involved the gradient and intercept. Similarly in part (iii) the calculation of v , using values of β and k found in part (ii), was more frequently incorrect than finding v by first using the graph to find the value of $\lg v$ corresponding to $\lg R = \lg 75$, although the value of $\lg v$ thus obtained was sometimes taken to be that of v itself.

Answers: (ii) 2.4 – 2.7, 1.55 – 1.65; (iii) 8.25 – 9.15.

Question 12 EITHER

This alternative was chosen by an overwhelming majority of candidates, many of whom obtained all or most of the marks available. In part (i) a few candidates used $fg(x)$ or, more rarely $g(x) \times f(x)$ and some evaluated $gf(2)$ but more frequently seen, especially from weaker candidates, was the misuse or absence of brackets in $2 - (3x - 2)$ leading to $2 - 3x - 2$ and $-3x$. Part (ii) was answered poorly in that, although $3x^2 - 8x + 8 = 0$ was obtained by most candidates, many could not continue to the correct conclusion. Some appeared to have reached a decision before considering the evidence by starting with ' $b^2 - 4ac > 0$ ' but were then puzzled on reaching ' $-32 > 0$ ', a statement which led some candidates to declare that there were 32, or even -32 , real roots. Others stated that, because it was a quadratic equation, there were 2 real roots, whilst quite often no comment at all was offered as to the number of real roots. Part (iii) was generally well done with only careless mistakes in manipulation a minor source of loss of marks. Part (iv) produced many correct solutions which often illustrated the idea of reflection in $y = x$. On the other hand there were some poor sketches in which non-uniform intervals were used along the axes, or in which there was clearly no appreciation that both f and f^{-1} are linear functions of x or, again, where the graph of $y = f^{-1}(x)$ passed through $(2, 0)$ and $(0, \frac{2}{3})$. Quite a number of candidates also omitted stating the coordinate of the point of intersection.

Answers: (i) $\frac{2}{3}$; (ii) 0; (iii) $\frac{x+2}{3}$, $2 - \frac{4}{x}$.

Question 12 OR

Weak candidates were usually unable to avoid error when re-arranging the terms in the required form, whereas good candidates coped easily; a few succeeded in solving by comparing coefficients. Part (ii) was rarely answered correctly in that although most candidates commented that the function was one-one, a property common to all functions with an inverse, very few explained how this applied to the particular function in the question. Part (iii) proved difficult for many; those starting from $x = 1 - y^2 + 6y$, rather than $x = 10 - (y - 3)^2$, were rarely able to make y the subject. Most candidates found the values -6 and 9 in part (iv), but only the stronger candidates could state the range correctly. The sketches offered in part (v) were usually the result of plotting points rather than an understanding of the shape of a quadratic curve with a maximum point. Consequently some curves were badly affected by a wayward point and some candidates simply used straight lines to join points. Those candidates producing a reasonable attempt almost always used the wrong curvature for that part of $y = g(x)$ reflected in the x -axis.

Answers: (i) $10, -3$; (iii) $3 + \sqrt{10 - x}$; (iv) $-6 < g(x) < 10$.