Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

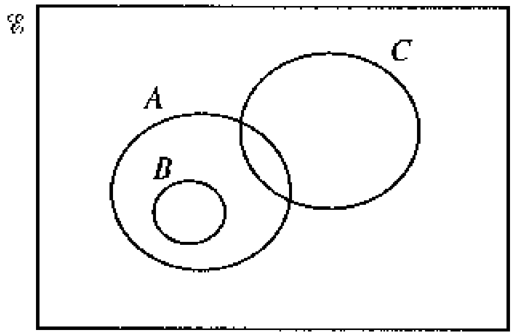
$$\Delta = \tfrac{1}{2} bc \sin A.$$

1 The line $4y = x + 11$ intersects the curve $y^2 = 2x + 7$ at the points A and B. Find the coordinates of the mid-point of the line AB. [4]

2 Show that $\cos\theta\left(\dfrac{1}{1-\sin\theta} - \dfrac{1}{1+\sin\theta}\right)$ can be written in the form $k\tan\theta$ and find the value of k. [4]

3 Solve the equation $\log_2 x - \log_4(x-4) = 2$. [4]

4

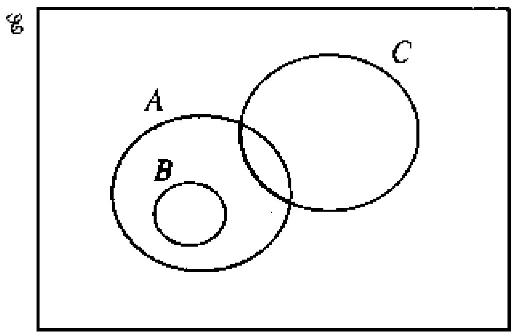


The diagram shows a universal set ℰ and the three sets A, B and C.

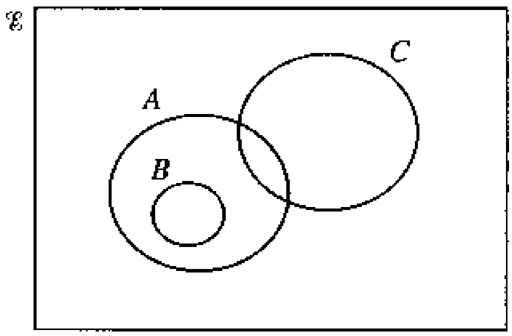
(i) Copy the above diagram and shade the region representing $(A \cup C) \cap B'$.

For each of the diagrams below, express, in set notation, the set represented hy the shaded area in terms of A, B and C.

(ii)



(iii)



[4]

[Turn over

5 Obtain

(i) the first 3 terms in the expansion, in descending powers of x, of $(3x - 1)^5$, [3]

(ii) the coefficient of x^4 in the expansion of $(3x - 1)^5(2x + 1)$. [2]

6 A particle travels in a straight line so that, t s after passing a fixed point A, its speed, v ms^{-1}, is given by

$$v = 40(e^{-t} - 0.1).$$

The particle comes to instantaneous rest at B. Calculate the distance AB. [6]

7 Given $\mathbf{A} = \begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ -2 & 3 \end{pmatrix}$, write down the inverse of **A** and of **B**. [3]

Hence find

(i) the matrix **C** such that $2\mathbf{A}^{-1} + \mathbf{C} = \mathbf{B}$, [2]

(ii) the matrix **D** such that $\mathbf{BD} = \mathbf{A}$. [2]

8 A garden centre sells 10 different varieties of rose bush. A gardener wishes to buy 6 rose bushes, all of different varieties.

(i) Calculate the number of ways she can make her selection. [2]

Of the 10 varieties, 3 are pink, 5 are red and 2 are yellow. Calculate the number of ways in which her selection of 6 rose bushes could contain

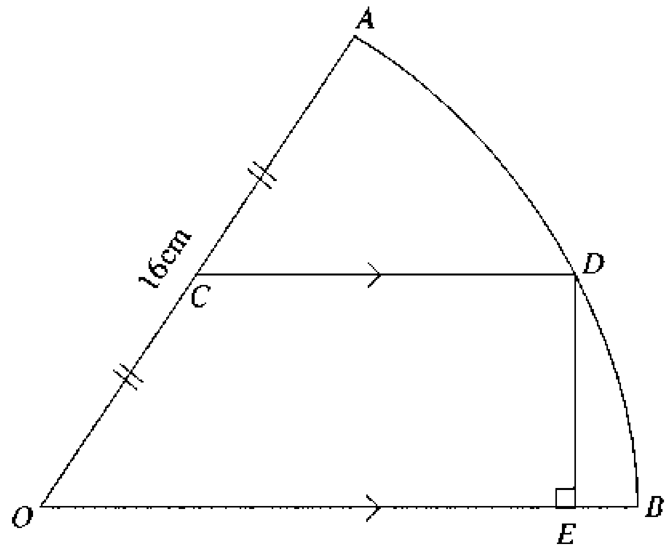
(ii) no pink rose bush, [1]

(iii) at least one rose bush of each colour. [4]

9 (i) Given that $y = (2x + 3)\sqrt{4x - 3}$, show that $\frac{dy}{dx}$ can be written in the form $\frac{kx}{\sqrt{4x - 3}}$ and state the value of k. [5]

(ii) Hence evaluate $\int_1^7 \frac{x}{\sqrt{4x - 3}}\,dx$. [3]

10



In the diagram, OAB is a sector of a circle, centre O and radius 16 cm, and the length of the arc AB is 19.2 cm. The mid-point of OA is C and the line through C parallel to OB meets the arc AB at D. The perpendicular from D to OB meets OB at E.

(i) Find angle AOB in radians. [2]

(ii) Find the length of DE. [2]

(iii) Show that angle DOE is approximately 0.485 radians. [2]

(iv) Find the area of the shaded region. [4]

11 A particle, moving in a certain medium with speed v m s^{-1}, experiences a resistance to motion of R N. It is believed that R and v are related by the equation $R = kv^{\beta}$, where k and β are constants.
The table shows experimental values of the variables v and R.

v	5	10	15	20	25
R	32	96	180	290	410

(i) Using graph paper, plot lg R against lg v and draw a straight line graph. [3]

Use your graph to estimate

(ii) the value of k and of β, [5]

(iii) the speed for which the resistance is 75 N. [2]

12 Answer only **one** of the following two alternatives.

EITHER

Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto 3x - 2, \quad x \neq \tfrac{4}{3},$$
$$\mathrm{g} : x \mapsto \frac{4}{2 - x}, \quad x \neq 2.$$

(i) Solve the equation $\mathrm{gf}(x) = 2$. [3]

(ii) Determine the number of real roots of the equation $\mathrm{f}(x) = \mathrm{g}(x)$. [2]

(iii) Express f^{-1} and g^{-1} in terms of x. [3]

(iv) Sketch, on a single diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, stating the coordinates of the point of intersection of the two graphs. [3]

OR

(i) Find the value of a and of b for which $1 - x^2 + 6x$ can be expressed in the form $a - (x + b)^2$. [3]

A function f is defined by $\mathrm{f} : x \mapsto 1 - x^2 + 6x$ for the domain $x \geqslant 4$.

(ii) Explain why f has an inverse. [2]

(iii) Find an expression for f^{-1} in terms of x. [2]

A function g is defined by $\mathrm{g} : x \mapsto 1 - x^2 + 6x$ for the domain $2 \leqslant x \leqslant 7$.

(iv) Find the range of g. [2]

(v) Sketch the graph of $y = |\mathrm{g}(x)|$ for $2 \leqslant x \leqslant 7$. [2]

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