UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

International General Certificate of Secondary Education

MARK SCHEME for the November 2004 question paper

0606 ADDITIONAL MATHEMATICS

0606/02 **Paper 2, maximum raw mark 80**

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2004 question papers for most IGCSE and GCE Advanced Level syllabuses.

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the November 2004 examination.

	maximum mark available	minimum mark required for grade:		
		A	C	E
Component 2	80	60	28	17

Grade A* does not exist at the level of an individual component.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW -1,2 This is deducted from A or B marks when essential working is omitted.

PA -1 This is deducted from A or B marks in the case of premature approximation.

S -1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX -1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

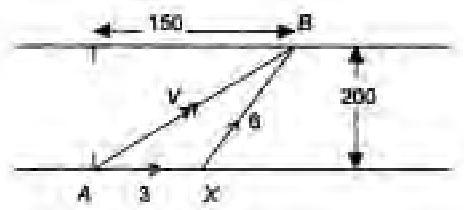
November 2004

INTERNATIONAL GCSE

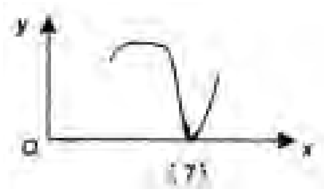
MARK SCHEME
MAXIMUM MARK: 80
SYLLABUS/COMPONENT: 0606/02 **ADDITIONAL MATHEMATICS** **PAPER 2**

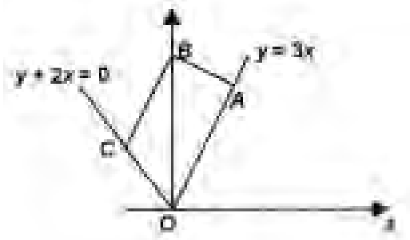
UNIVERSITY of CAMBRIDGE
International Examinations

1 **[4]**	$\mathbf{A}^{-1}=\begin{pmatrix}4 & -3\\5 & 2\end{pmatrix}\times\frac{1}{23}$	B1	B1
	$\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{23}\begin{pmatrix}4 & -3\\5 & 2\end{pmatrix}\begin{pmatrix}-4\\-13\end{pmatrix}=\begin{pmatrix}1\\-2\end{pmatrix}$	M1	A1
2 **[4]**	$\frac{13}{4+\sqrt{3}}\times\frac{4-\sqrt{3}}{4-\sqrt{3}}=4-\sqrt{3}$ or $\left(\frac{13}{4+\sqrt{3}}\right)^2=\frac{169}{19+8\sqrt{3}}$	M1	A1
	$\left(4-\sqrt{3}\right)^2=19-8\sqrt{3}$ or $\frac{169}{19+8\sqrt{3}}\times\frac{19-8\sqrt{3}}{19-8\sqrt{3}}=19-8\sqrt{3}$	M1	A1
	OR $\left(a+b\sqrt{3}\right)\left(19+8\sqrt{3}\right)=169\Rightarrow\begin{cases}19b+8a=0\\19a+24b=169\end{cases}$ solve M1 $\Rightarrow\left.\begin{matrix}a=19\\b=-8\end{matrix}\right\}$ A1		
3 **[5]**	Integrate $-3/2\cos 2x$ $+ 4\sin x$	M1 A1	A1
	$[\ ]_0^{\pi/2}=5.5-(-1.5)=7$ Must use both limits properly, not assume cos0 = 0, not use $\pi/2$ degrees.	M1	A1
4 **[5]**	Eliminate $y\to(x+2)^2+(x+k)^2$ or $x\to x^2+(y-2+k)^2$	M1	
	$2x^2+(4+2k)x+(2+k^2)=0$ or $2y^2+(2k-4)y+(k^2-4k+2)=0$		
	Apply "b^2-4ac" $\Rightarrow 16k-4k^2$	M1	A1
	$\Rightarrow\begin{matrix}k=0\text{ or }4\\0\le k\le 4\end{matrix}$ **OR** $\begin{cases}k\ge 0 & \text{B1}\\k\le 4 & \text{B1}\end{cases}$	M1 A1	
	Solving quadratic in k to 2 solutions – condone <		
5 **[6]**	$\log_4(3x)+\log_4(0.5)=\log_4(1.5x)$	B1	
	$\log_{16}(3x-1)=\frac{\log_4(3x-1)}{\log_4 16}$ For change of base – also to base 10,16, 2	M1	
	½ $\log_4(3x-1)=\log_4\sqrt{3x-1}$ or $2\log_4(1.5x)=\log_4(2.25x^2)$ Changing $k\log z$ to $\log z^k$	M1	
	$3x-1=2.25x^2$	A1	
	$9x^2-12x+4=0$ $\Rightarrow$ $(3x-2)^2=0$ $\Rightarrow$ $x=\frac{2}{3}$	M1	A1
	Solving 3 term quadratic Accept 0.66 or 0.67 or better		

6 [6]	**(i)** $3\sin\theta - 2\cos\theta = 3\cos\theta + 2\sin\theta \Rightarrow \sin\theta = 5\cos\theta \Rightarrow \tan\theta = 5$ **OR**, squaring + Pythagoras $\Rightarrow \sin\theta = 5/\sqrt{26}$ or $\cos\theta = 1/\sqrt{26}$ for M1 $\theta = 78.7°$ or 1.37 rad or better (acute angle *only* accepted) **(ii)** $x^2 + y^2 = (9\sin^2\theta - 12\sin\theta\cos\theta + 4\cos^2\theta) + (9\cos^2\theta + 12\sin\theta\cos\theta + 4\sin^2\theta)$ $= 13\sin^2\theta + 13\cos^2\theta$ = 13 Pythagoras	M1 A1 A1 B1 M1 A1 c.s.o
7 [7]	Put $x = a \Rightarrow 6a^3 + 5a^2 - 12a = -4$ or divide by $x - a$ to remainder Search $6(-2)^3 + 5(-2)^2 = 12(-2) + 4 = 0 \Rightarrow a = -2$ (at least 2, if unsuccessful, for M1) similarly, if $a = \frac{1}{2}$ or $\frac{2}{3}$ is found $6a^3 + 5a^2 - 12a + 4 \equiv (a + 2)(6a^2 - 7a + 2)$ **OR**, finding 2nd root $6a^2 - 7a + 2 \equiv (3a - 2)(2a - 1) = 0 \Rightarrow a = \frac{1}{2}, \frac{2}{3}$ **OR**, finding 3rd root	M1 M1 A1 M1 A1 M1 A1
8 [7]	 $BAX = \tan^{-1} 200/150 = \tan^{-1} 4/3 \approx 53.13°$, or 36.87°, or 250 $ABX = \sin^{-1}\{(3\sin BAX) \div 6\} = \sin^{-1} 0.4 \approx 23.58°$ Incorrect obtuse-angled Δs – allow M1 for use of sine or cosine rule) $AXB = 180° - (53.13° + 23.58°) = 103.29°$ $V = (6\sin 103.29°) \div \sin 53.13° = 7.3$ [or via cosine rule] [or $V_{ACROSS} = 6\sin 76.71° \approx 5.84$ or $V_{DOWN} = 3 + 6\cos 76.71° \approx 4.38$] Time = 250 ÷ 7.3 [or 200 ÷ 5.8 or 150 ÷ 4.4] ≈ 34 s (accept 34 ~ 34.5) 2 stages can be combined by applying cosine rule to velocities: $36 = V^2 + 9 - 6V\cos 53.13°$ M1 $\Rightarrow 10V^2 - 36V - 270 = 0$ A1 Solve M1 $V = 2.3$ A1 3 stages can be combined by applying cosine rule to displacements: $(6t)^2 = (250)^2 + (3t)^2 - 6t\cos 53.13°$ M2 $\Rightarrow 27t^2 + 900t - 62500 = 0$ A2 Solve DM1 $t = 34.3$ A1 First 5 marks by vector method: $V = (150\mathbf{i} + 200\mathbf{j})/t$ B1 $V_{BOAT} = (150\mathbf{i} + 200\mathbf{j})/t - 3\mathbf{i}$ M1A1	 B1 M1 A1 M1 A1 DM1 A1

	$\lvert V_{BOAT} \rvert = \lvert (150 - 3t)\mathbf{i}/t + 200\mathbf{j}/t \rvert = 6$ M1 $\Rightarrow 27t^2 + 900t - 62500 = 0$ A1 By scale drawing: Construct 53.13 with 200, 150   $\Rightarrow$ Velocity Δ M1A1 $\Rightarrow V = 7.3 \pm 0.1$ M1A1 $\Rightarrow T = 34$ DM1A1	
9 **[7]**	**(i)** $Y = \log y$, $X = x$ $m = \log b$, $c = \log a$ **(ii)** $Y = \log y$, $X = \log x$ $m = k$, $c = \log A$ **(iii)** $Y = 1/y$, $X = 1/x$ $\begin{cases} c = 1/p \\ m = -9/p \end{cases}$ [Other valid alternatives acceptable Y y y/x x x/y $1/x$ X y/x y x/y x $1/y$ m q $1/q$ p $1/p$ $-p/q$ c p $-p/q$ q $-q/p$ $1/q$]	B1 DB1 B1 DB1 M1 A1 A1

10 **[9]**	**(i)**	Let $y = x^2 - 8x + 7$ $dy/dx = 2x - 8 = 0$ at $x = 4$	M1
		$d^2y/dx^2 = 2$ $\therefore$ min at $x = 4$	A1
		OR via completing the square: $y = (x - 4)^2 - 9 \Rightarrow$ min -9 at $x = 4$	
		$\therefore$ f(x) has maximum at $x = 4$, corroborated by argument re reflection of –9 or by graph	B2, 1, 0
	(ii) 		B2, 1, 0
		Judge by shape, unless values clearly incorrect. Ignore curve outside domain. Cusp needed at x-axis. Accept straight line for right-hand arm, but curvature, if shown, must be correct.	
	(iii)	$0 \le f(x) \le 9$ [condone <]	B1 B1
	(iv)	$k = 4$	B1

11 **[10]**		
	Let *A* be (*x*, *y*) i.e. (*x*, 3*x*)	B1
	Length of *OA* = $\sqrt{x^2+9x^2}=\sqrt{250} \Rightarrow x = 5$, *A* is (5, 15) ($\sqrt{x^2+y^2}=\sqrt{250}$ enough for M1)	M1 A1
	Gradient of *AB* is $-\frac{1}{3}$	B1
	Equation of *AB* is $y - 15 = -\frac{1}{3}(x-5) \Rightarrow$ *B* is $(0, 16\frac{2}{3})$ **AND** substitute *x* = 0 for M1 Decimals 16.6 or 16.7, – 1 p.a.	M1 A1
	Gradient of *BC* is 3	B1
	Equation of *BC* is $y = 3x + 16\frac{2}{3}$	M1
	Meets $y + 2x = 0$ when $-2x = 3x + 16\frac{2}{3} \Rightarrow x = -3\frac{1}{3}$,	M1
	C is $(-3\frac{1}{3}, 6\frac{2}{3})$ but accept (–3.32, 6.64), (–3.34, 6.68)	A1
	In essence, scheme is 3 marks for each of *A, B, C*. Possible to find *B* before *A* e.g. $A\hat{O}X = \tan^{-1}3 = 71.565^\circ$ B1 $OB = \sqrt{250}/\sin 71.565^\circ$ M1 $\Rightarrow 16\frac{2}{3}$ A1 Gradient of *AB* is $-\frac{1}{3}$ B1 Solve $y - 16\frac{2}{3} = -\frac{1}{3}x$ with $y = 3x$ M1 $\Rightarrow$ (5,15) A1	
12 **[10]** **EITHER**	**(i)** $v = \int a\,\mathrm{d}t = 1.4t - 0.3t^2 + 0.5$	M1 A2,1,0
	At rest $v = 0 \Rightarrow 3t^2 - 14t - 5 = 0 \Rightarrow (3t+1)(t-5) = 0 \Rightarrow t = 5$ **OR**, by verifying $[1.4t - 0.3t^2 + 0.5]_{t=5} = 0$	A1
	(ii) $s = \int v\,\mathrm{d}t = 0.7t^2 - 0.1t^3 + 0.5t$	M1 A1√
	$[s]_{t=5} = 7.5$	A1√
	$[s]_{t=10} = -25$ **OR** $s_{10} - s_5 = -32.5$ Total distance = $(2 \times 7.5) + 25 = 40$ **OR** 7.5 + 32.5	A1M1A1$_{\text{c.s.o.}}$

12 [10] OR	**(i)** $\int y\,dx = \int\left(3x + \frac{2}{x^2}\right)dx = \frac{3x^2}{2} - \frac{2}{x}$ accept $\frac{ax^2}{2} - \frac{b}{x}$ One term correct sufficient for M1	M1 A1
	$[\ ]_2^4 = \left(24 - \frac{1}{2}\right) - (6 - 1) = 18.5$	DM1 A1
	(ii) (2, 3) on curve $\Rightarrow 3 = 2a + \frac{b}{4}$	B1
	$\frac{dy}{dx} = a - \frac{2b}{x^3}$ $\left[\frac{dy}{dx}\right]_{x=2} = 0 \Rightarrow a - \frac{b}{4} = 0$	M1 M1
	Solving $a = 1$, $b = 4$	A1
	$y = x + \frac{4}{x^2} \Rightarrow \frac{dy}{dx} = 1 - \frac{8}{x^3} \Rightarrow \frac{d^2y}{dx^2} = \frac{24}{x^4} > 0$ when $x = 2$ $\therefore$ min [or any equivalent method]	M1 $A1_{c.s.o}$