Momentum

Momentum (ρ)

- Every moving object has momentum.
- Linear momentum is defined as the product of mass and velocity of an object.

momentum (kg m/s) = mass (kg) × velocity (m/s)

$\rho = mv$

- Momentum is a vector quantity and has the direction of velocity.

Newton's second law of motion states that:

The rate of change of momentum is directly proportional to the unbalanced force acting on that body and takes place in the same direction.

Hence in terms of momentum Newton's second law is:

$$F_{net} = \frac{\Delta p}{\Delta t}$$

If mass stays constant , then the above equation can be re-written as:

$$F_{net} = \frac{\Delta p}{\Delta t} = \frac{p_{final} - p_{initial}}{\Delta t} = \frac{mv_{final} - mv_{initial}}{\Delta t} = \frac{m(v_{final} - v_{initial})}{\Delta t} = \frac{m\Delta v}{\Delta t}$$

$$F_{net} = ma$$

Numerical:

A ball of mass 540 g moving vertically downwards hits the ground with a velocity of 8.0 m s^{-1}. After impact it bounces upwards with an initial velocity of 5.0 m s^{-1}.

a Calculate the momentum of the ball immediately before and after impact.

b What was the change of momentum during impact (Δp)?

a Initial momentum before impact, $p = mu = 0.54 \times 8.0 = 4.3$ kg m s^{-1} downwards. Final momentum after impact, $p = mv = 0.54 \times 5.0 = 2.7$ kg m s^{-1} upwards.

The opposite directions may be represented by positive and negative signs rather than written descriptions. That is, the momentum before impact was +4.3 kg m s^{-1} and the momentum afterwards was −2.7 kg m s^{-1}. (The choice of signs is interchangeable.)

b $\Delta p = (-2.7) - (+4.3) = -7.0$ kg m s^{-1} (upwards)

Example:

A bullet with a mass of 0.03 kg leaves a gun at 1000 m/s. If the gun's mass is 1.5 kg, what is the velocity of the recoil on the gun?

momentum of bullet = mass × velocity

= 0.03 kg × 1,000 m/s

= 30 kg m/s

Rearrange the equation:

velocity = momentum ÷ mass

velocity of recoil on gun = 30 kg m/s ÷ 1.5 kg

= 20 m/s

Impulse

- If you get hit by a ball, the effect is greater if it bounces off you than if you catch it. This is because the change in momentum is greater if the ball bounces off you.
- Unit of impulse=kgm/s
- Impulse is a vector

Example:

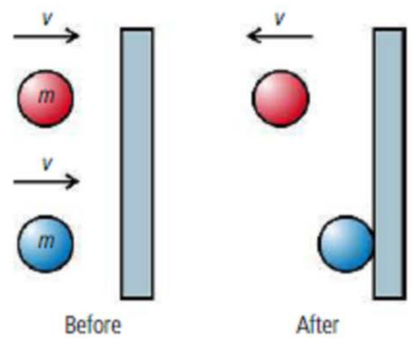
Red ball

Momentum before = mv
Momentum after = $-mv$ (remember momentum is a vector)
Change in momentum = $-mv - mv = -2mv$

Blue ball

Momentum before = mv
Momentum after = 0
Change in momentum = $0 - mv = -mv$

The impulse is defined as the change of momentum.



The change of momentum of the red ball is greater.

Solved examples from past papers:

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7 An object of mass 50 kg accelerates from a velocity of 2.0 m / s to a velocity of 10 m / s in the same direction.

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What is the impulse provided to cause this acceleration?

A 250 Ns **B** 400 Ns **C** 850 Ns **D** 2500 Ns

Explanation:
Change on velocity= (10-2)=8 m/s
mass =50kg
Hence the impulse that caused this acceleration
=F= m x (Change in velocity)= 50 x 8 =400Ns

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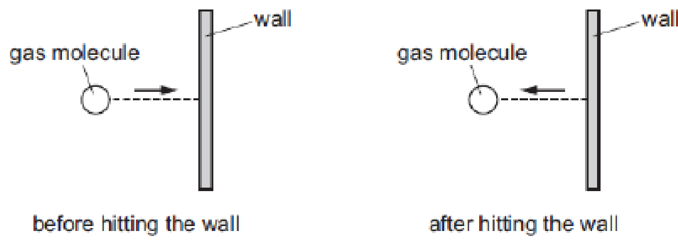
8 A gas molecule strikes the wall of a container. The molecule rebounds with the same speed.



What happens to the kinetic energy and what happens to the momentum of the molecule?

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	kinetic energy	momentum
A	changes	changes
B	changes	stays the same
C	stays the same	changes
D	stays the same	stays the same

Answer =C

Explanation:

Since there is no change in the speed, so the kinetic energy stays the same. The momentum changes because the direction of motion and hence the velocity changes. So the momentum which is a product of mass and velocity changes too.

Principle of momentum

Statement:

The total linear momentum of any system is constant, provided that no external forces are acting on it.

The following points should be kept in mind while using the principle of momentum:

- Momentum is a vector quantity so its direction must always be included in calculations.
- The system must be isolated- only the interacting objects must be considered and there can be no forces acting on that system from outside.
- Immediately after an interaction, external forces like friction will usually affect the motion of objects.

In real lives:

- Loss of momentum will happen because the system is not isolated as the system will be acted upon by external forces.
- Some or all momentum may appear to be lost when something collides with an object that has a much greater mass. The motion after impact may be too small to observe or measure .Example: a person jumping on the earth's surface. The predicted motion of the " person-earth system" is insignificant after impact.
- The force of gravity generally increases the momentum of falling objects. But the objects are not in isolated systems, there are external forces acting on them. For example: a 3kg rock experiences a gravitational force towards the earth of approximately 30N and therefore gains momentum as it accelerates downwards. The law of conservation of momentum clearly predicts that the earth must gain an equal momentum upwards towards the rock. Because the mass of the earth is so large, its gain of momentum is insignificant.

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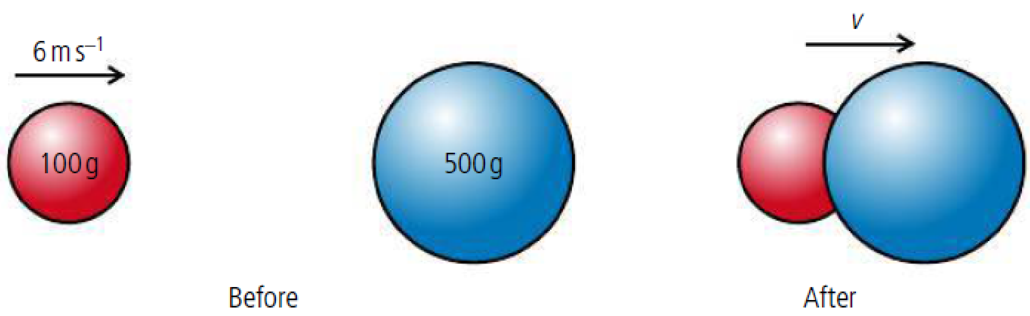
1. A collision where the bodies join together

If two balls of modelling clay collide with each other they stick together as shown in Figure 2.85. We want to find the velocity, v, of the combined lump after the collision.



If bodies are isolated then momentum is conserved so:

$$\text{momentum before} = \text{momentum after}$$

$$0.1 \times 6 + 0.5 \times 0.0 = 0.6 \times v$$

$$v = \frac{0.6}{0.6} = 1\ \text{m s}^{-1}$$

2. An explosion

A ball of clay floating around in space suddenly explodes into a big piece and a small piece, as shown in Figure 2.86. If the big bit has a velocity of $5\ \text{m s}^{-1}$, what is the velocity of the small bit?

Since this is an isolated system, momentum is conserved so:

$$\text{momentum before} = \text{momentum after}$$

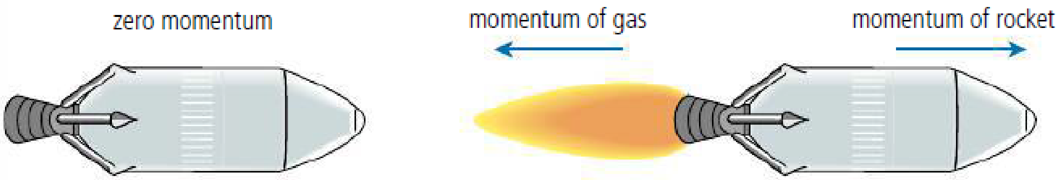
$$0 \times 0.12 = 0.02 \times (-v) + 0.1 \times 5$$

$$0.02 \times v = 0.5$$

$$v = 25\ \text{m s}^{-1}$$

Rocket engine

The momentum of a rocket plus fuel floating in space is zero but when the engines are fired gas is expelled at high speed. This gas has momentum towards the left so in order to conserve momentum the rocket will move towards the right.



The momentum of the rocket will equal the momentum of the expelled gases, so increasing the rate at which the gases are expelled will increase the acceleration of the rocket. Note that the situation for a rocket about to blast off on the Earth is rather more complex since the rocket + fuel can no longer be considered isolated.

Jet engine

A jet engine produces thrust (the force that pushes the plane forwards) by increasing the speed of air taken in at the front by passing it through a series of turbines. The fast-moving air expelled from the back of the engine has an increased momentum so if momentum is to be conserved the plane must have increased momentum in the forwards direction.

Numerical

Mass A (4.0 kg) is moving at 3.0 m s^{-1} to the right when it collides with mass B (6.0 kg) moving in the opposite direction at 5.0 m s^{-1}.

a If they stick together after the collision, what is their velocity?

b What assumption did you make?

a The total momentum must be the same before and after collision. Choosing velocities and momenta (momentums) to the right to be positive and to the left to be negative:

momentum of A = $m_A u_A = 4.0 \times (+3.0) = +12.0\,\text{kg m s}^{-1}$

momentum of B = $m_B u_B = 6.0 \times (-5.0) = -30\,\text{kg m s}^{-1}$

Therefore, the total momentum (before) the collision is +12 +(−30) = −18 kg m s^{-1}. The total momentum after the collision must be the same as that before, so:

$m_{AB} v_{AB} = -18\,\text{kg m s}^{-1}$

$(4 + 6) v_{AB} = -18\,\text{kg m s}^{-1}$

$v_{AB} = \frac{-18}{10} = -1.8\,\text{m s}^{-1}$

The negative sign means the velocity is to the left.

This explanation has been written out in detail to aid understanding. A more direct way of answering any such question involving two masses interacting is as follows:

momentum before interaction = momentum after interaction

$m_A u_A + m_B u_B = m_A v_A + m_B v_B$

In this example:

$(4.0 \times 3.0) + (6.0 \times -5.0) = (4.0 + 6.0) \times v_{AB}$

$v_{AB} = -1.8\,\text{m s}^{-1}$

b The assumption made is that there are no external forces acting on the system. If there are significant frictional forces involved, the calculated answer can be taken to be the instantaneous velocity immediately after collision, and the effects of friction can be considered afterwards.

APPLICATION BASED QUESTIONS:

MCQ

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8 A girl of mass 50 kg runs at 6.0 m/s.

What is her momentum?

A 300 J **B** 300 kg m/s **C** 900 J **D** 900 kg m/s

8 A moving body undergoes a change of momentum.

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What is a unit for change of momentum?

A N m **B** N/m **C** N s **D** N/s

9 A ball of mass 2.0 kg is travelling at a speed of 12 m/s. It moves towards an object of mass 3.0 kg which is at rest.

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The ball hits the object and sticks to it.

Which row gives the total momentum, and the speed of both objects immediately after the collision?

	total momentum kg m/s	speed m/s
A	0	4.8
B	0	8.0
C	24	4.8
D	24	8.0

EXPENDED THEORY

2 Fig. 2.1 shows a hammer being used to drive a nail into a piece of wood.

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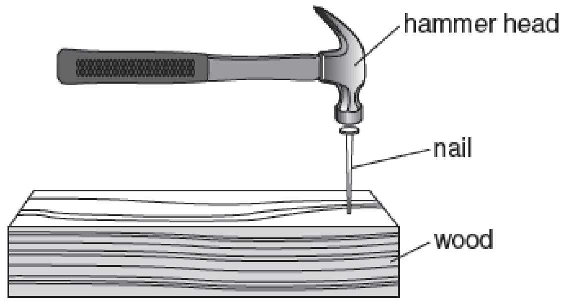

Fig. 2.1

The mass of the hammer head is 0.15 kg.
The speed of the hammer head when it hits the nail is 8.0 m/s.
The time for which the hammer head is in contact with the nail is 0.0015 s.

The hammer head stops after hitting the nail.

(a) Calculate the change in momentum of the hammer head.

change in momentum = ...[2]

(b) State the impulse given to the nail.

impulse = ...[1]

(c) Calculate the average force between the hammer and the nail.

average force = ...[2]

[Total: 5]

2 **(a)** Explain why momentum is a vector quantity. **F/M/2017-P42**

....[1]

(b) The crumple zone at the front of a car is designed to collapse during a collision.

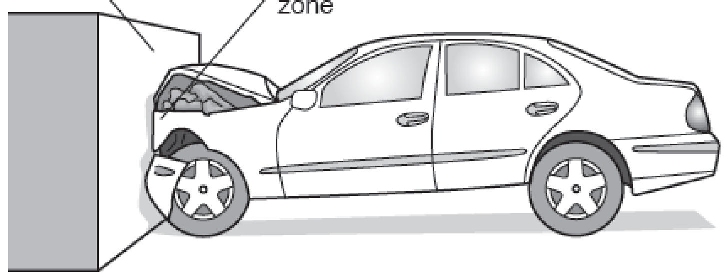

Fig. 2.1

In a laboratory test, a car of mass 1200 kg is driven into a concrete wall, as shown in Fig. 2.1.

A video recording of the test shows that the car is brought to rest in 0.36 s when it collides with the wall. The speed of the car before the collision is 7.5 m/s.

Calculate

(i) the change of momentum of the car,

change of momentum = ...[2]

(ii) the average force acting on the car.

average force = ...[2]

(c) A different car has a mass of 1500 kg. It collides with the same wall and all of the energy transferred during the collision is absorbed by the crumple zone.

(i) The energy absorbed by the crumple zone is 4.3×10^5 J. Show that the speed of the car before the collision is 24 m/s.

[2]

(ii) Suggest what would happen to the car if it is travelling faster than 24 m/s when it hits the wall.

....

....[1]

[Total: 8]

3 (a) Underline the pair of quantities which must be multiplied together to calculate *impulse*.

force and mass	force and velocity	mass and time
time and velocity	weight and velocity	force and time

[1]

(b) Fig. 3.1 shows a collision between two blocks A and B on a smooth, horizontal surface.

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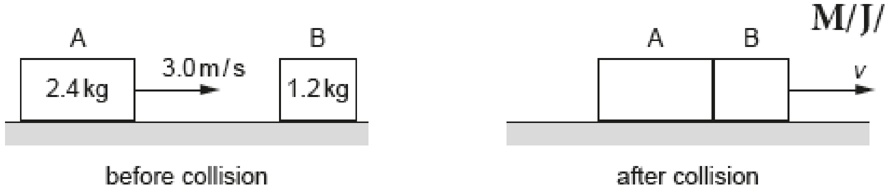

Fig. 3.1

Before the collision, block A, of mass 2.4 kg, is moving at 3.0 m/s. Block B, of mass 1.2 kg, is at rest.

After the collision, blocks A and B stick together and move with velocity *v*.

(i) Calculate

1. the momentum of block A before the collision,

momentum = ...[2]

2. the velocity *v*,

velocity = ...[2]

3. the impulse experienced by block B during the collision.

impulse = ...[2]

(ii) Suggest why the total kinetic energy of blocks A and B after the collision is less than the kinetic energy of block A before the collision.

....

....[1]

[Total: 8]

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4 A balloon contains a fixed mass of gas.

(a) Explain, in terms of the momentum of molecules, how the gas in the balloon exerts a pressure.

....

....

....

....[2]