Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME: SOLVED BY SMART EDU HUB

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2816863460

MATHEMATICS **0580/41**

Paper 4 (Extended) **May/June 2018**

2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator, Geometrical instruments, Tracing paper (optional)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
If working is needed for any question it must be shown below that question.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.

This document consists of **19** printed pages and **1** blank page.

DC (SC/CGW) 147701/2

1 Adele, Barbara and Collette share \$680 in the ratio 9 : 7 : 4.

(a) Show that Adele receives \$306.

Adele receives: $\frac{9}{9+7+4} \times 680 = 306$

Adele \$ 306

(b) Calculate the amount that Barbara and Collette each receives.

Barbara receives: $\frac{7}{9+7+4} \times 680 = 238$

Collette receives: $\frac{4}{9+7+4} \times 680 = 136$

Barbara \$ 238

Collette \$ 136 [3]

(c) Adele changes her \$306 into euros (€) when the exchange rate is €1 = \$1.125 .
Calculate the number of euros she receives.

€1 = \$1.125
€ x =\$306
Hence we get:
1.125x=306
So, x= 306/1.125=€272

€ 272 [2]

(d) Barbara spends a total of \$17.56 on 5 kg of apples and 3 kg of bananas.
Apples cost \$2.69 per kilogram.

Calculate the cost per kilogram of bananas.

Let number of apples be "a" and bananas be "b"

Hence: 5a +3b=17.56

So, 5(2.69) +3b=17.56

Hence b= $\frac{17.56-5(2.69)}{3} = 1.37$

\$ 1.37 [3]

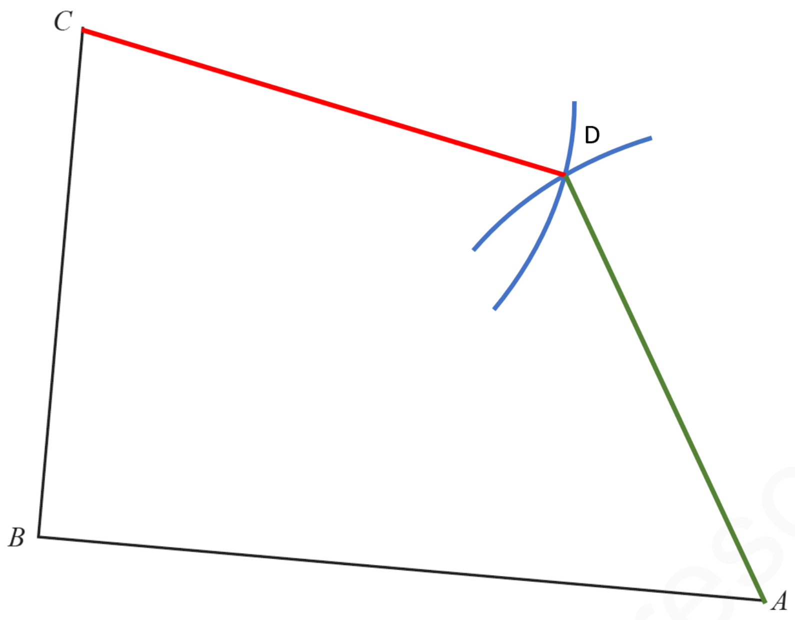
(e) Collette spends half of her share on clothes and $\frac{1}{5}$ of her share on books.
Calculate the amount she has left.

Amount left with Collette is:

136-136($\frac{1}{2}+\frac{1}{5}$) $= 40.8$

\$ 40.8 [3]

2 The scale drawing shows two boundaries, *AB* and *BC*, of a field *ABCD*.
The scale of the drawing is 1 cm represents 8 m.



Hint: Draw arcs of 9cm from C and A respectively

Scale: 1 cm to 8 m

(a) The boundaries *CD* and *AD* of the field are each 72 m long.

(i) Work out the length of *CD* and *AD* on the scale drawing.

Given:
1cm=8m
For CD and AD each,
Length on the scale drawing
=72/8=9cm

9
.... cm [1]

(ii) **Using a ruler and compasses only**, complete accurately the scale drawing of the field. [2]

(b) A tree in the field is

This kind of construction is not there in the syllabus

- equidistant from *A* and *B*

and

- equidistant from *AB* and *BC*.

On the scale drawing, construct two lines to find the position of the tree.
Use a straight edge and compasses only and leave in your construction arcs. [4]

3 **(a)** The price of a house decreased from $82 500 to $77 500.

Calculate the percentage decrease.

Percentage decrease= $\frac{Old\ price - New\ price}{Old\ price} \times 100$

$= \frac{82500-77500}{82500} \times 100 = 6.06$

6.06

.... % [3]

(b) Roland invests $12 000 in an account that pays compound interest at a rate of 2.2% per year.

Calculate the value of his investment at the end of 6 years.
Give your answer correct to the nearest dollar.

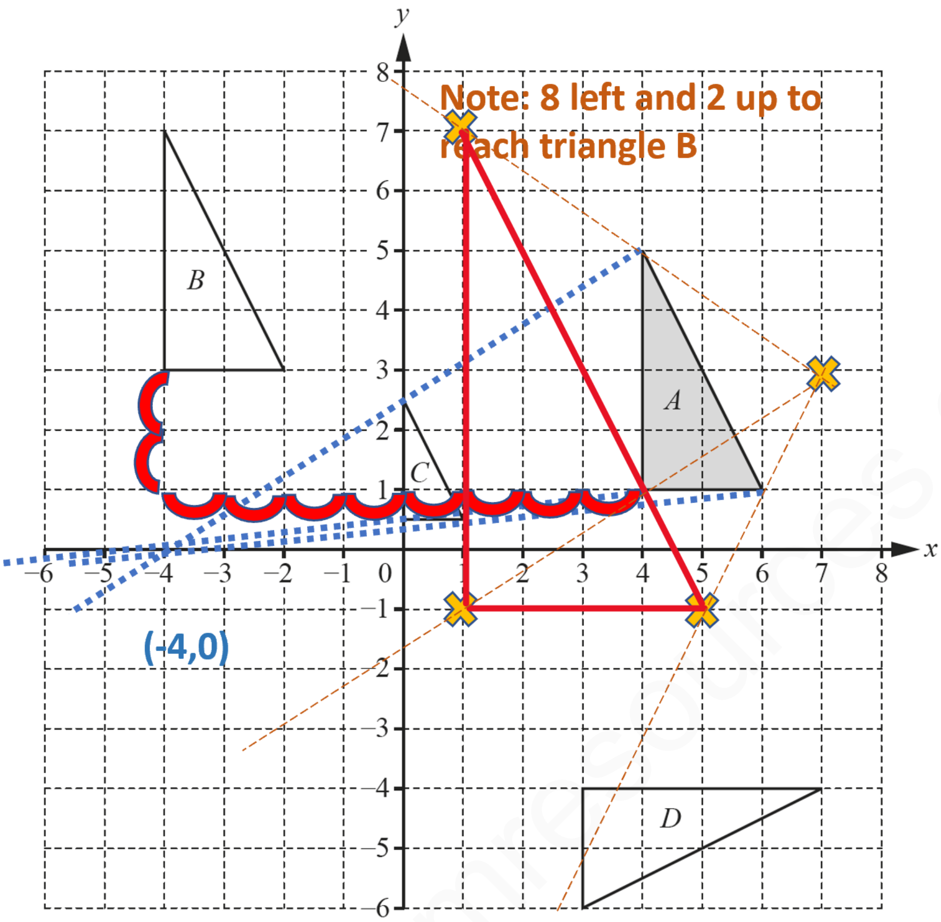
P=$12000
R=2.2% per year
T= 6 years
Value of his investment=Amount=P$(1+\frac{R}{100})^N$

$=12000(1+\frac{2.2}{100})^6$

=$ 13674

$ .... 13674 [3]

4



(a) Describe fully the **single** transformation that maps

(i) triangle A onto triangle B,

Translation $\begin{pmatrix} -8 \\ 2 \end{pmatrix}$

....

.... [2]

(ii) triangle A onto triangle C,

Enlargement, sf=0.5, Centre of enlargement (-4,0)

....

.... [3]

(iii) triangle A onto triangle D.

Rotation 90^0 Clockwise [This can be easily found using a tracing paper]

....

.... [3]

(b) On the grid, draw the image of triangle A after an enlargement by scale factor 2, centre $(7, 3)$. [2]

5 **(a)** Factorise.

(i) $2mn + m^2 - 6n - 3m$

$2mn + m^2 - 6n - 3m$
$= m(2n+m) - 3(2n+m)$
$= (m-3)(2n+m)$

(m-3)(2n+m)
.... [2]

(ii) $4y^2 - 81$

$4y^2 - 81$
$= (2y)^2 - (9)^2$

$= (2y-9)(2y+9)$

(2y-3)(2y+3)
.... [1]

(iii) $t^2 - 6t + 8$

$t^2 - 6t + 8$
$= t^2 - 2t - 4t + 8$
$= t(t-2) - 4(t-2)$
$= (t-4)(t-2)$

(t-2)(t-4)
.... [2]

(b) Rearrange the formula to make x the subject.

$$k = \frac{2m - x}{x}$$

$k = \frac{2m-x}{x}$
$kx = 2m - x$
$kx + x = 2m$
$x(k+1) = 2m$
$x = \frac{2m}{k+1}$

$x = \frac{2m}{k+1}$
$x =$ .... [4]

(c) Solve the simultaneous equations. You must show all your working.

$$\frac{1}{2}x - 3y = 9$$
$$5x + y = 28$$

$\frac{1}{2}x - 3y = 9 \dots \dots . . > Equation\ 1$

5x+y=28 $\dots \dots . . > Equation\ 2$

Rearranging equation 1:

$x - 6y = 18$

X= 6y+18........> Equation1

Substitute x= 6y+18 in equation 2:

5x+y=28

Hence:

5(6y+18)+y=28

30y+90 +y=28

31y=28-90

31y=-62

Hence: y= $\frac{-62}{31} = -2$

Also:5x+y=28...Equation-> 2

Hence 5x-2=28

5x=30

X=6

Hence x=6 and y=-2

$x =$ x=6

$y =$ y=-2 [3]

(d) $$\frac{3}{m+4} - \frac{4}{m} = 6$$

(i) Show that this equation can be written as $6m^2 + 25m + 16 = 0$.

$$\frac{3}{m+4} - \frac{4}{m} = 6$$

Hence:

3m-4(m+4) =6(m)(m+4)

3m-4m-16=6m²+24

-m=16+6m²+24m

16+6m²+24m+m=0

6m²+25m+16=0

[3]

(ii) Solve the equation $6m^2 + 25m + 16 = 0$.

Show all your working and give your answers correct to 2 decimal places.

Comparing the equation:

6m²+25m+16 with

am²+bm+c we get:

a=6 , b=25 and c=16

Hence:

Applying the quadratic formula we get:

m=$\frac{-25\pm\sqrt{25^2-4(6)(16)}}{2(6)}$

Plugging in the calculator we get;

m=-0.79

Or

m= -3.38

$m =$ -0.79 or $m =$ -3.38 [4]

6 A solid hemisphere has volume 230 cm^3.

(a) Calculate the radius of the hemisphere.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

Volume of a hemisphere= 230cm^3

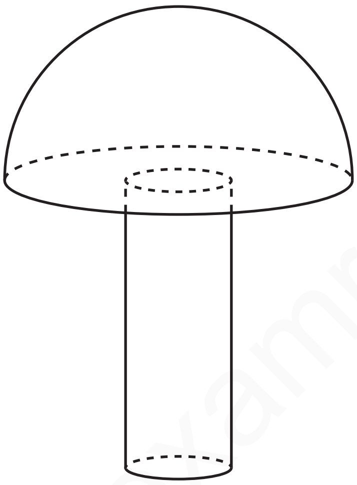
Volume of a sphere =V=$\frac{4}{3}\pi r^3$

Volume of a hemisphere V=230 $= \frac{4}{6}\pi r^3$

Hence: r=$\sqrt[3]{\frac{230 \times 6}{4\pi}} = \sqrt[3]{\frac{230 \times 3}{2\pi}}$ = 4.79cm

4.79 .... cm [3]

(b) A solid cylinder with radius 1.6 cm is attached to the hemisphere to make a toy.



NOT TO SCALE

The total volume of the toy is 300 cm^3.

(i) Calculate the height of the cylinder.

Total volume of the toy= volume of hemisphere+ volume of the cylinder

300=$\frac{4}{6}\pi r^3 + \pi r^2 h$

$300 = 230 + \pi r^2 h$ [Volume of cylinder: Already given]

300-230= $\pi(1.6)^2 h$

h=$\frac{70}{(\pi)1.6^2}$=8.70cm

.... cm [3]

(ii) A mathematically similar toy has volume 19 200 cm^3.

Calculate the radius of the cylinder for this toy.

Relation between volume of silimilar objects and their radii is:

$\frac{V_1}{V_2}=(\frac{r_1}{r_2})^3$

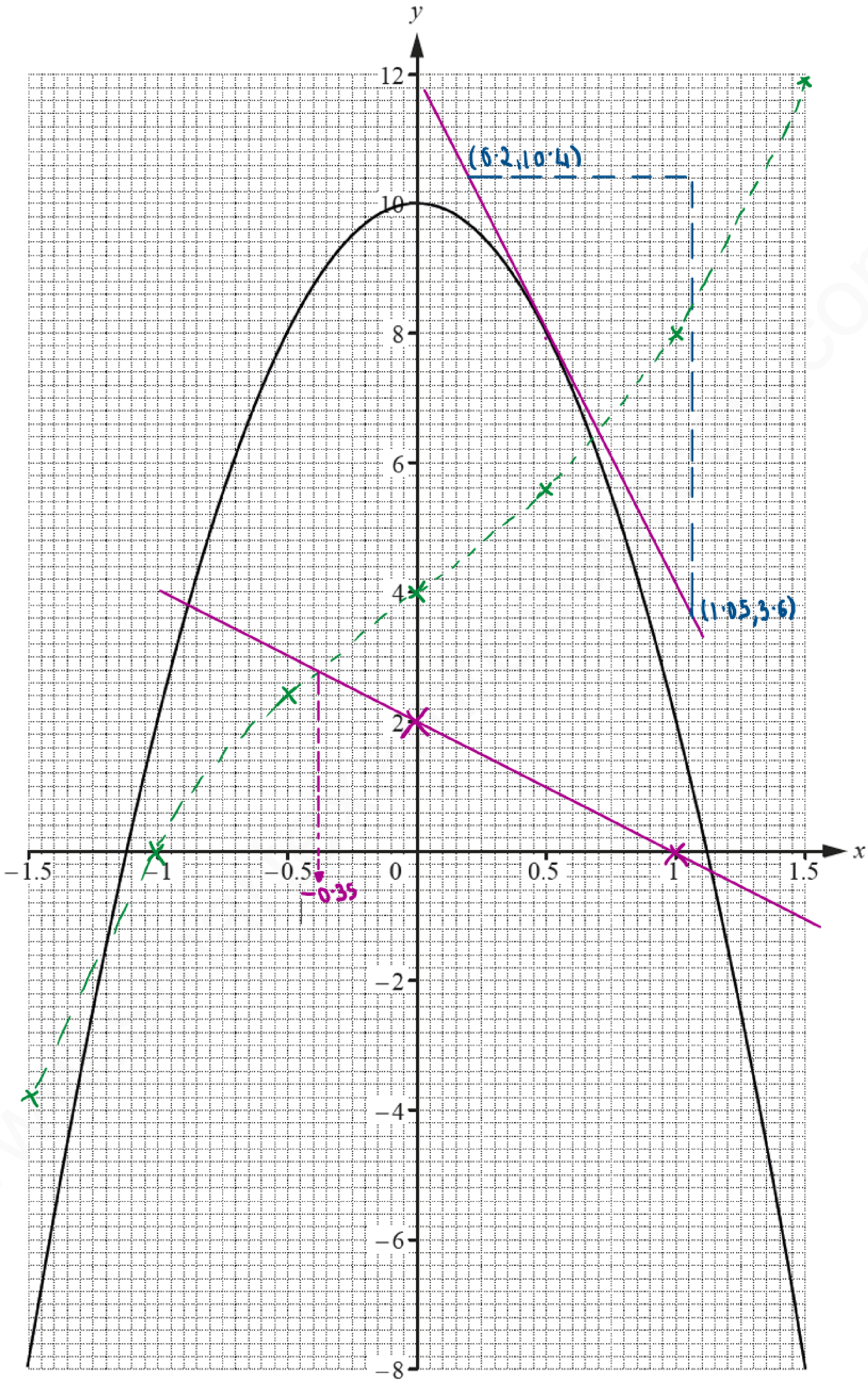
Hence:

$\frac{19200}{300}=(\frac{r_1}{1.6})^3$

$r_1=\sqrt[3]{\frac{19200}{300}}\times 1.6 = 6.4$

6.4

.... cm [3]


y
12
10
8
6
4
2
−2
−4
−6
−8
x
−1.5
−1
−0.5
0
0.5
1
1.5
(0·2, 10·4)
(1·05, 3·6)
−0·35

(a) Write down the equation of the line of symmetry of the graph.

x=0

.... [1]

(b) On the grid opposite, draw the tangent to the curve at the point where $x = 0.5$.
Find the gradient of this tangent.

$$\frac{10 \cdot 4 - 3 \cdot 6}{0 \cdot 2 - 1.05} = -8$$

-8

.... [3]

(c) The table shows some values for $y = x^3 + 3x + 4$.

x	-1.5	-1	-0.5	0	0.5	1	1.5
y	-3.9	0	2.4	4	5.6	8	11.9

(i) Complete the table. [3]

(ii) On the grid opposite, draw the graph of $y = x^3 + 3x + 4$ for $-1.5 \leqslant x \leqslant 1.5$. [4]

(d) Show that the values of x where the two curves intersect are the solutions to the equation $x^3 + 8x^2 + 3x - 6 = 0$.

Equating the equations of the two curves we get: $10-8x^2=x^3+3x+4$
Rearranging the equation we get:
$10-4-8x^2-x^3-3x=0$
$6-8x^2-x^3-3x=0$, which can be further rewritten as below:
$x^3+8x^2+3x-6=0$

[1]

(e) By drawing a suitable straight line, solve the equation $x^3 + 5x + 2 = 0$ for $-1.5 \leqslant x \leqslant 1.5$.

Draw the line y=-2x+2
[Because x^3+3x+4 can be rewritten as
$x^3+3x+2x-2x+2+2$]

x	0	1
y =-2x+2	2	0

$x =$ -0.35 .... [3]

8 **(a)** The exterior angle of a regular polygon is $x°$ and the interior angle is $8x°$.

Calculate the number of sides of the polygon.

exterior angle+ interior angle of a polygon=180^0

x+8x=180

9x=180

x=20

Hence: Exterior angle=20^0 and interior angle=8x=160^0

Value of each interior angle=$\frac{(n-2)\times 180}{n} = 160$

Hence: 180n-360=160n

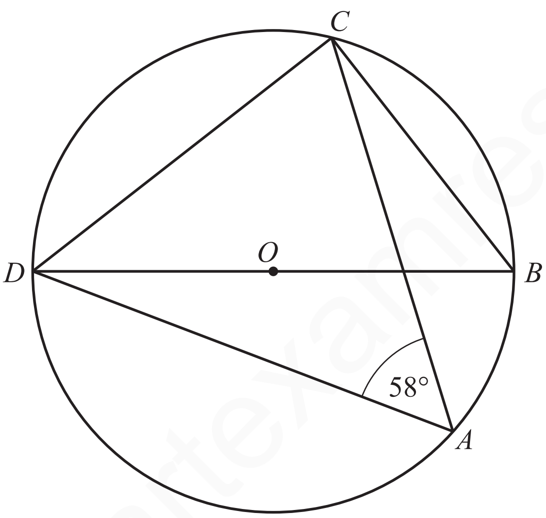
-360=-20n

n=$\frac{360}{20}$=18

Hence the polygon has 18 sides

18 .... [3]

(b)



NOT TO SCALE

A, *B*, *C* and *D* are points on the circumference of the circle, centre *O*.
DOB is a straight line and angle *DAC* = 58°.

Find angle *CDB*.

∠DBC= ∠DAC = 58^0 [Angles in the same segment]

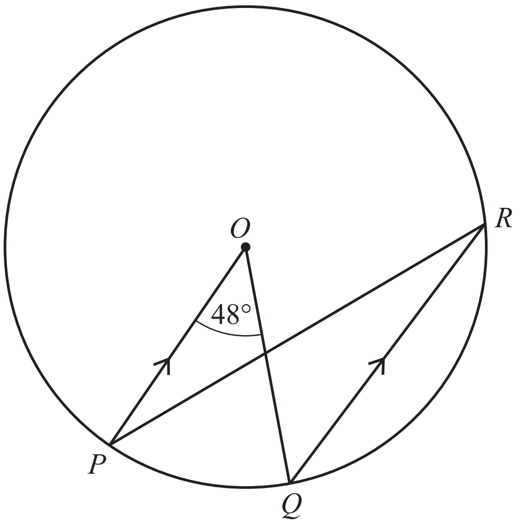
∠DCB = 90^0 [Angles in a semicircle]

Hence:

∠CDB= 180-(∠DCB +∠DBC)= 180-(90+58)=32^0

Angle *CDB* = 32^0 .... [3]

(c)



P, *Q* and *R* are points on the circumference of the circle, centre *O*.
PO is parallel to *QR* and angle *POQ* = 48°.

(i) Find angle *OPR*.

∠POQ= ∠OQR =48^0 [Alternate angles]
∠POQ= 2 x ∠PRQ[Central angle is twice the angle at the circumference]
Hence: ∠PRQ=$\frac{48}{2}$=24^0
Also;
∠OPR= ∠PRQ = [Alternate angles]
Hence
∠OPR=24^0

24
.... [2]

(ii) The radius of the circle is 5.4 cm.

Calculate the length of the **major** arc *PQ*.

Angle of the major arc= 360^0-the central angle= 360^0-48^0= 312^0
Hence ;
Length of the major arc=$\frac{\theta}{360} \times 2\pi r = \frac{312}{360} \times 2\pi(5.4) = 29.4$

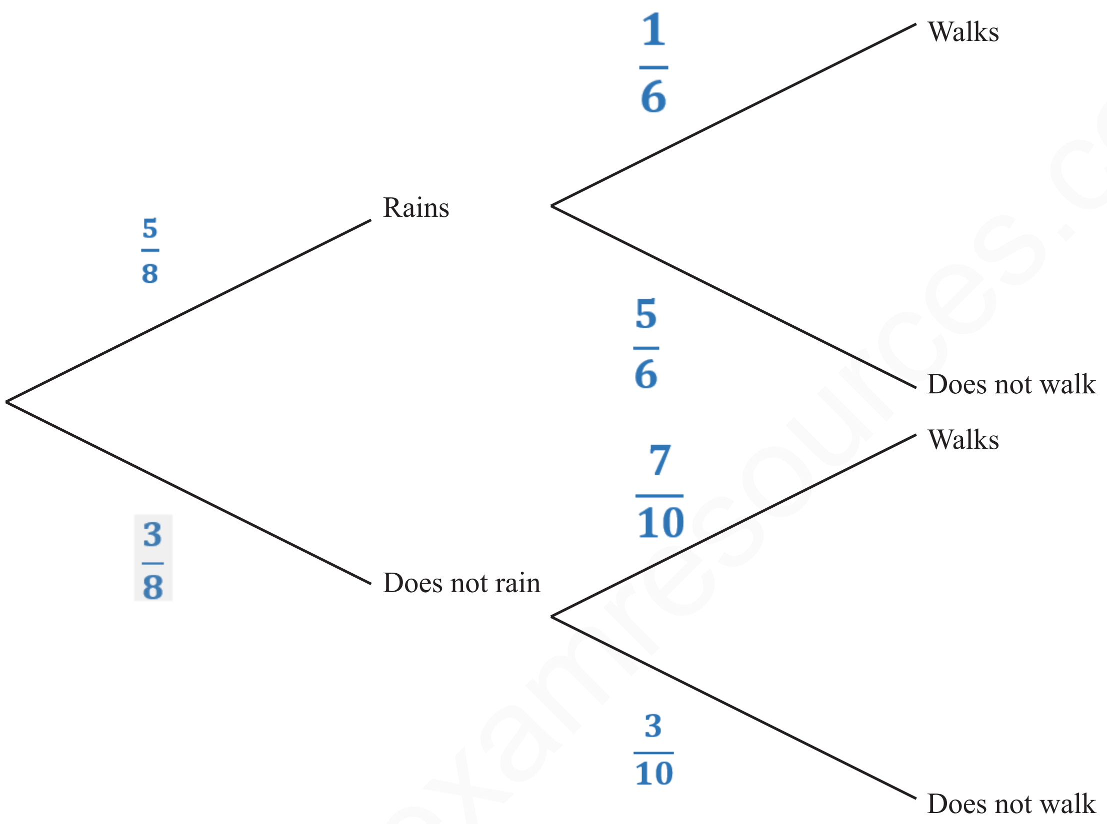
29.4
.... cm [3]

9 The probability that it will rain tomorrow is $\frac{5}{8}$.

If it rains, the probability that Rafael walks to school is $\frac{1}{6}$.

If it does not rain, the probability that Rafael walks to school is $\frac{7}{10}$.

(a) Complete the tree diagram.



[3]

(b) Calculate the probability that it will rain tomorrow and Rafael walks to school.

$$\frac{5}{8} \times \frac{1}{6} = \frac{5}{48}$$

$\frac{5}{48}$ .... [2]

(c) Calculate the probability that Rafael does not walk to school.

Note: This includes the probability that he does not walk to school, irrespective of whether it rains or not.

$$\frac{5\times5}{6\times8} + \frac{3\times3}{8\times10} = \frac{25}{48} + \frac{9}{80} = \frac{304}{480}$$

$\frac{304}{480}$ .... [3]

10 **(a)** In 2017, the membership fee for a sports club was $79.50. This was an increase of 6% on the fee in 2016. Calculate the fee in 2016.

Let the membership fee in 2016 before increase be $x

Given that:

$$\left(\frac{6}{100} \times x\right) + x = 79.50$$

$$\left(\frac{6x}{100}\right) + x = 79.50$$

$$\frac{106x}{100} = 79.50$$

106x=7950

$X=\frac{7950}{106}=75$

Hence the fee in 2016 was $75

$ ....75.... [3]

(b) On one day, the number of members using the exercise machines was 40, correct to the nearest 10. Each member used a machine for 30 minutes, correct to the nearest 5 minutes.

Calculate the lower bound for the number of minutes the exercise machines were used on this day.

One member used the machine for 30 minutes correct to 10 minutes. Hence :

$Time_{LB}$=27.5minutes

$Time_{UB}$=32.5minutes

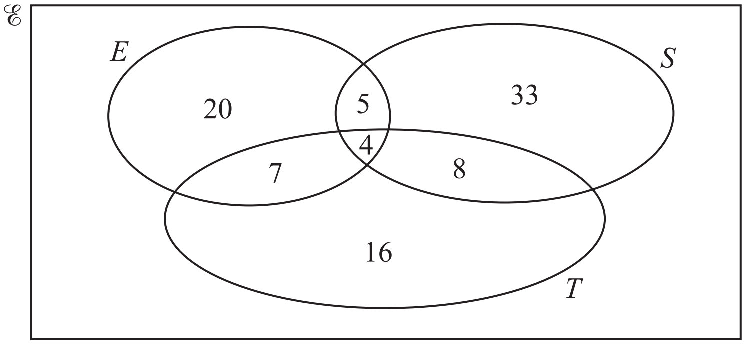
Total members that used the exercise machine=40

Hence $Members_{LB}$= 35

Hence the Total $time_{LB}$ that the exrcise machine got used on this day= 35 x 27.5=962.5

....962.5.... min [2]

(c) On another day, the number of members using the exercise machines (E), the swimming pool (S) and the tennis courts (T) is shown on the Venn diagram.



(i) Find the number of members using only the tennis courts.

Note:It the that part of Set T that exclusdes all the overlapping regions

16
.... [1]

(ii) Find the number of members using the swimming pool.

Method: 5+4+8+33=50
Note:[Add all memebers in the Swimming pool circle]

50
.... [1]

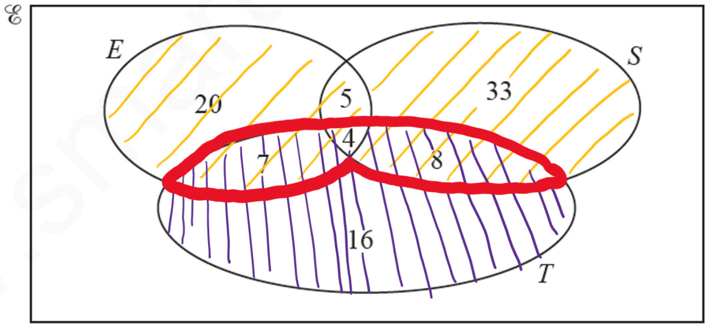
(iii) A member using the swimming pool is chosen at random.
Find the probability that this member also uses the tennis courts and the exercise machines.

Note:
This refers to those members that use all the three, hence it the the intersection region of all the three circles

4
--
50
.... [2]

(iv) Find $n(T \cap (E \cup S))$.

Note: The following duagram is not needed. It is only to aid your understanding of the concept



The required region has 7+4+8=19 members as shown above by the region bordered in red

19
---------------------[1]

11 **(a)** $\overrightarrow{OA} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $\overrightarrow{AB} = \begin{pmatrix} 8 \\ -7 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$

Find

(i) $|\overrightarrow{OB}|$,

Distance formula

$$= \begin{pmatrix} 4+8 \\ 3-7 \end{pmatrix} = \begin{pmatrix} 12 \\ -4 \end{pmatrix} = \sqrt{12^2 + (-4)^2} = 12.6$$

$|\overrightarrow{OB}|$ = **12.6** .... [3]

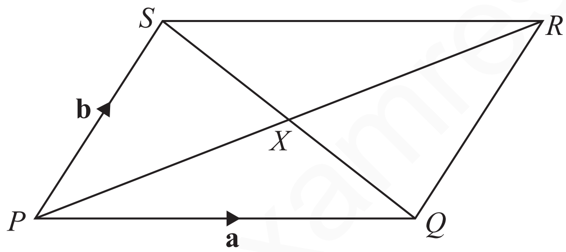
(ii) $\overrightarrow{BC}$.

$$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$$

$$= \begin{pmatrix} -8 \\ 7 \end{pmatrix} + \begin{pmatrix} -3 \\ 6 \end{pmatrix} = \begin{pmatrix} -11 \\ 13 \end{pmatrix}$$

$\overrightarrow{BC} = \begin{pmatrix} -11 \\ 13 \end{pmatrix}$ [2]

(b)



NOT TO SCALE

PQRS is a parallelogram with diagonals *PR* and *SQ* intersecting at *X*.
$\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{PS} = \mathbf{b}$.

Find $\overrightarrow{QX}$ in terms of **a** and **b**.
Give your answer in its simplest form.

Note that the diagonals of a parallelogram bisect each other.

This implies:

QX=XS

QX=$\frac{1}{2}$QS

QS=-a+b. Hence; QX= $\frac{1}{2}(\boldsymbol{b} - \boldsymbol{a})$

$\overrightarrow{QX}$ = $\frac{1}{2}(\boldsymbol{b} - \boldsymbol{a})$ .... [2]

(c) $\mathbf{M} = \begin{pmatrix} 2 & 5 \\ 1 & 8 \end{pmatrix}$

Calculate

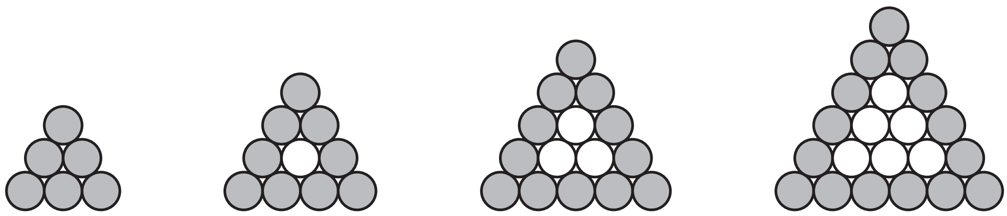
(i) $\mathbf{M}^2$,

$\mathbf{M}^2 = \begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(ii) $\mathbf{M}^{-1}$.

$\mathbf{M}^{-1} = \begin{pmatrix} & \\ & \end{pmatrix}$ [2]

12 Marco is making patterns with grey and white circular mats.



Pattern 1 Pattern 2 Pattern 3 Pattern 4

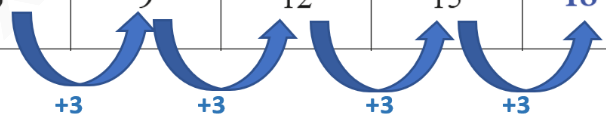
The patterns form a sequence.
Marco makes a table to show some information about the patterns.

Pattern number	1	2	3	4	5
Number of grey mats	6	9	12	15	18
Total number of mats	6	10	15	21	28

(a) Complete the table for Pattern 5. [2]

(b) Find an expression, in terms of *n*, for the number of grey mats in Pattern *n*.

Pattern number	1	2	3	4	5
Number of grey mats	6	9	12	15	18



The above is a linear sequence as its first difference is constant. Hence its nth term is given by:

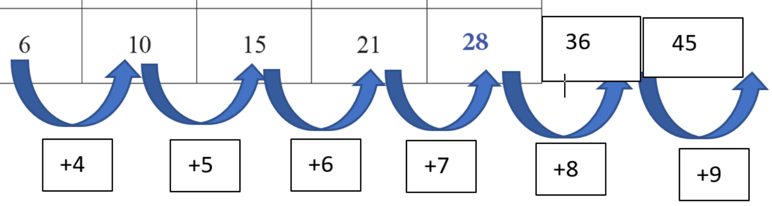
$$t_n = a + (n-1)d = 6 + (n-1)3 = 6 + 3n - 3 = 3n + 3$$

3n+3 .... [2]

(c) Marco makes a pattern with 24 grey mats.

3n+3=24
Hence 3n=21
Hence: n=7

Pattern number	1	2	3	4	5
Number of grey mats	6	9	12	15	18
Total number of mats	6	10	15	21	28



45 .... [2]

(d) Marco needs a total of 6 mats to make the first pattern.
He needs a total of 16 mats to make the first two patterns.
He needs a total of $\frac{1}{6}n^3 + an^2 + bn$ mats to make the first n patterns.

Find the value of a and the value of b.

$$For\ Pattern\ 1;\ \frac{1}{6}(1)^3 + a(1)^2 + b(1) = \frac{1}{6} + a + b = 6 - -\!\!\!\longrightarrow\ Equation\ 1$$

$$For\ Pattern\ 2;\ \frac{1}{6}(2)^3 + a(2)^2 + b(2) = \frac{8}{6} + 4a + 2b = 16 - -\!\!\!\longrightarrow\ Equation\ 2$$

Simplifying equation 1 and 2 we get;

$6a + 6b = 35 - -\!\!\!\longrightarrow Equation - 3$

And

$24a + 12b = 88 - -\!\!\!\longrightarrow Equation\ 4$

Multiplying equation 3 by 2 we get:

12a + 12b = 70— — —→Equation-5

Solving Equation 3 and equation 5 simultaneously we get:

$24a + 12b = 88 - -\!\!\!\longrightarrow \quad Equation\ 4$

-[12a + 12b = 70] $- -\!\!\!\longrightarrow \quad Equation\ 5$

---

12 a = 18

a= $\frac{18}{12} = \frac{3}{2}$

Substituting value of a in Equation 3 we get:

6a+6b=35

Hence:

$6\left(\frac{3}{2}\right) + 6b = 35$

6b= 35-9

6b=26

b= $\frac{26}{6} = \frac{13}{3}$

$a =$ 3/2

$b =$ 13/3 [6]

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