FUNCTIONS-SET-4-QP-MS

1 The function f is defined, for $0 < x < \pi$, by $f(x) = 5 + 3\cos 4x$. Find

(i) the amplitude and the period of f, [2]

(ii) the coordinates of the maximum and minimum points of the curve $y = f(x)$. [4]

MARKING SCHEME

	$f(x) = 5 + 3\cos 4x$		
(i)	$a = 3$, period $= \frac{1}{2}\pi$	B1 B1	Co. allow 90° for period.
(ii)	max/min $x = \pi/4$ or $2\pi/4$ or $3\pi/4$ $\rightarrow$ max of 8 $\rightarrow$ min of 2	 B1 B1	 When "8" is used as stationary value. When "2" is used as stationary value.
	$(\pi/4, 2)$ $(2\pi/4, 8)$ $(3\pi/4, 2)$	B2, 1√	√ for 5 ± his "a". [B0 if degrees here]
		[6]	Ignore inclusion of max/min at 0 or π.

2

The function f is defined, for $x \in \mathbb{R}$, by

$$f : x \mapsto \frac{3x + 11}{x - 3}, \quad x \neq 3.$$

(i) Find f^{-1} in terms of x and explain what this implies about the symmetry of the graph of $y = f(x)$. [3]

The function g is defined, for $x \in \mathbb{R}$, by

$$g : x \mapsto \frac{x - 3}{2}.$$

(ii) Find the values of x for which $f(x) = g^{-1}(x)$. [3]

(iii) State the value of x for which $gf(x) = -2$. [1]

MARKING SCHEME

(i)	$y = (3x + 11)/(x - 3)$		
	Makes x the subject.	M1	Good algebra in making x the subject.
	$f^{-1}(x) = (3x + 11)/(x - 3)$	A1	
	f and f^{-1} are the same functions.		
	→ Graph has $y = x$ as line of symmetry.	B1	Co accept any mention of $y = x$.
		[3]	
(ii)	$g(x) = ½(x - 3)$ $g^{-1}(x) = 2x + 3$	B1	Anywhere.
	→ $2x + 3 = (3x + 11)/(x - 3)$ → $2x^2 - 6x - 20 = 0$ → $x = -2$ or 5	M1 A1	Algebra must lead to quadratic. Co.
		[3]	
(iii)	$gf(x) = -2$ → $f(x) = g^{-1}(-2)$ → $x = -2$	B1	However obtained.
		[1]	

3

The function f is defined by $\mathrm{f}(x) = 2 - \sqrt{x+5}$ for $-5 \leqslant x < 0$.

(i) Write down the range of f. [2]

(ii) Find $\mathrm{f}^{-1}(x)$ and state its domain and range. [4]

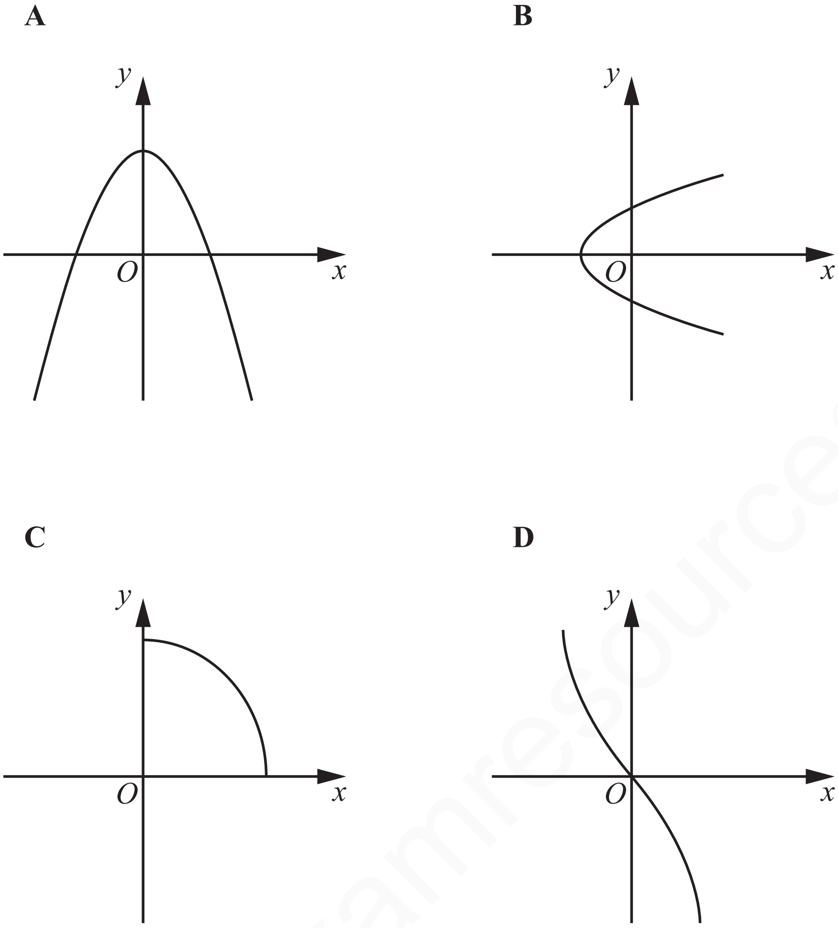
The function g is defined by $\mathrm{g}(x) = \dfrac{4}{x}$ for $-5 \leqslant x < -1$.

(iii) Solve $\mathrm{fg}(x) = 0$. [3]

MARKING SCHEME

(i)	$2-\sqrt{5} < \mathrm{f}(x) \leqslant 2$	**B2**	**B1** for $\leqslant 2$ **B1** for $2-\sqrt{5}<$ or awrt -0.24 Must be using f, f(x) or y, $2-\sqrt{5}<$, if not then **B1** max
(ii)	$\mathrm{f}^{-1}(x) = (2-x)^2 - 5$ Domain $2-\sqrt{5} < x \leqslant 2$ Range y or $-5 \leqslant \mathrm{f}^{-1}(x) < 0$	**M1** **A1** **B1** **B1**	for a correct method to find the inverse Must be using the correct variables for the B marks
(iii)	$\mathrm{fg}(x) = \mathrm{f}\left(\frac{4}{x}\right)$ $= 2 - \sqrt{\frac{4}{x} + 5}$ leading to $x = -4$	**M1** **DM1** **A1**	for correct order of functions for solution of equation

4 Diagrams **A** to **D** show four different graphs. In each case the whole graph is shown and the scales on the two axes are the same.



Place ticks in the boxes in the table to indicate which descriptions, if any, apply to each graph. There may be more than one tick in any row or column of the table. [4]

	A	B	C	D
Not a function				
One-one function				
A function that is its own inverse				
A function with no inverse				

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<tr><th>A</th><th>B</th><th>C</th><th>D</th></tr>
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<td>B1 for either each row correct or each column correct – mark to candidate's advantage.</td>
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5 **(a)** Functions f and g are such that, for $x \in \mathbb{R}$,

$$f(x) = x^2 + 3,$$

$$g(x) = 4x - 1.$$

(i) State the range of f. [1]

(ii) Solve $fg(x) = 4$. [3]

(b) A function h is such that $\mathrm{h}(x) = \dfrac{2x+1}{x-4}$ for $x \in \mathbb{R},\ x \neq 4$.

(i) Find $\mathrm{h}^{-1}(x)$ and state its range. [4]

(ii) Find $\mathrm{h}^{2}(x)$, giving your answer in its simplest form. [3]

MARKING SCHEME

(a)(i)	$f \geqslant 3$	**B1**	must be using a correct notation
(a)(ii)	$(4x-1)^2+3=4$	**M1**	correct order
	solution of resulting quadratic equation	**DM1**	
	$x=0,\ x=\frac{1}{2}$	**A1**	both required
(b)(i)	$xy-4y=2x+1$	**M1**	'multiplying out'
	$x(y-2)=4y+1$ $x=\frac{4y+1}{y-2}$	**M1**	collecting together like terms
	$h^{-1}(x)=\frac{4x+1}{x-2}$	**A1**	correct answer with correct notation
	Range $h^{-1} \neq 4$	**B1**	must be using a correct notation
(b)(ii)	$h^2(x)=h\left(\frac{2x+1}{x-4}\right)$ $=\frac{2\left(\frac{2x+1}{x-4}\right)+1}{\left(\frac{2x+1}{x-4}\right)-4}$	**M1**	dealing with h^2 correctly
	dealing with fractions within fractions	**M1**	
	$=\frac{5x-2}{17-2x}$ oe	**A1**	

6

Functions f and g are defined, for $x > 0$, by

$$f(x) = \ln x,$$
$$g(x) = 2x^2 + 3.$$

(i) Write down the range of f. [1]

(ii) Write down the range of g. [1]

(iii) Find the exact value of $f^{-1}g(4)$. [2]

(iv) Find $g^{-1}(x)$ and state its domain. [3]

MARKING SCHEME

(i)	$y \in \mathbb{R}$ oe	**B1**	Must have correct notation i.e. no use of x
(ii)	$y > 3$ oe	**B1**	Must have correct notation i.e. no use of x
(iii)	$f^{-1}(x) = e^x$ or $g(4) = 35$	**B1**	First B1 may be implied by correct answer or by use of 35
	$f^{-1}g(4) - e^{35}$	**B1**	
(iv)	$\frac{y-3}{2} = x^2$ or $\frac{x-3}{2} = y^2$	**M1**	valid attempt to obtain the inverse
	$g^{-1}(x) = \sqrt{\frac{x-3}{2}}$	**A1**	correct form, must be $g^{-1}(x) =$ or $y =$
	Domain $x > 3$	**B1**	Must have correct notation

7

(a) It is given that

$$f : x \mapsto \sqrt{x} \qquad \text{for} \quad x \geqslant 0,$$
$$g : x \mapsto x+5 \qquad \text{for} \quad x \geqslant 0.$$

Identify each of the following functions with one of f^{-1}, g^{-1}, fg, gf, f^2, g^2.

(i) $\sqrt{x+5}$ [1]

(ii) $x-5$ [1]

(iii) x^2 [1]

(iv) $x+10$ [1]

(b) It is given that $h(x) = a + \dfrac{b}{x^2}$ where a and b are constants.

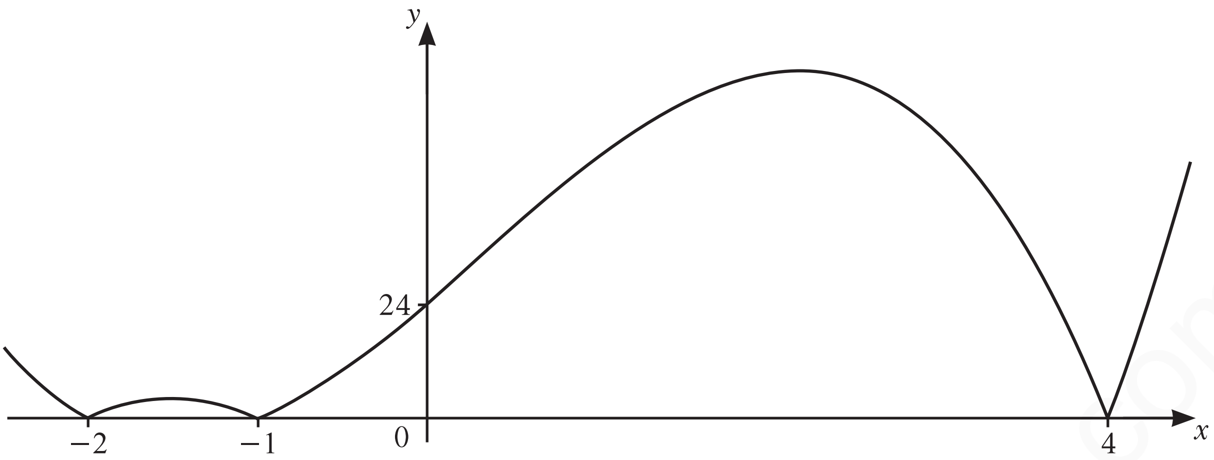
(i) Why is $-2 \leqslant x \leqslant 2$ not a suitable domain for $h(x)$? [1]

(ii) Given that $h(1) = 4$ and $h'(1) = 16$, find the value of a and of b. [2]

MARKING SCHEME

a(i)	fg	**B1**	
a(ii)	g^{-1}	**B1**	
a(iii)	f^{-1}	**B1**	
a(iv)	g^2	**B1**	
(b)(i)	Undefined at $x = 0$ oe	**B1**	
(b)(ii)	$4 = a + b$ $h'(x) = \frac{p}{x^3}$ and attempt at $h'(1)$	**M1**	For attempt at $h(1)$ and differentiation to obtain $h'(1)$, must have the form $h'(x) = \frac{p}{x^3}$ oe
	$b = -8$ $a = 12$	**A1**	For both

1



The diagram shows the graph of $y = |p(x)|$, where $p(x)$ is a cubic function. Find the two possible expressions for $p(x)$. [3]

	$y = \pm 3(x+2)(x+1)(x-4)$	3	**B1** for 3 **B1** for $(x+2)(x+1)(x-4)$ **B1** for $\pm$