QUADRATIC FUCNTIONS

2.13 The functions f and g are defined for real values of x by

$$f(x) = \sqrt{x-1} - 3 \qquad \text{for } x > 1,$$

$$g(x) = \frac{x-2}{2x-3} \qquad \text{for } x > 2.$$

(i) Find gf(37). [2]

(ii) Find an expression for $f^{-1}(x)$. [2]

(iii) Find an expression for $g^{-1}(x)$. [2]

(i) $\quad$ f$(37)=3$ or gf$(x)=\dfrac{\sqrt{x-1}-3-2}{2(\sqrt{x-1}-3)-3}$ $\qquad$ **B1**

$\quad$ gf$(37)=\dfrac{3-2}{6-3}=\dfrac{1}{3}$ $\qquad$ **B1**

(ii) $\quad y=\sqrt{x-1}-3 \rightarrow (y+3)^2=x-1$ $\qquad$ **M1** $\qquad$ Rearrange and square in any order

$\quad (x+3)^2+1=$ f$^{-1}(x)$ oe isw $\qquad$ **A1** $\qquad$ Interchange x and y and complete

(iii) $\quad y=\dfrac{x-2}{2x-3}$

$\quad 2xy-3y=x-2 \rightarrow 2xy-x=3y-2$ $\qquad$ **M1** $\qquad$ Multiply and collect like terms

$\quad \dfrac{3x-2}{2x-1}=$ g$^{-1}(x)$ oe $\qquad$ **A1** $\qquad$ Interchange and complete
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ Mark final answer

2.14

The functions f and g are defined for real values of x by

$$f(x) = \frac{2}{x} + 1 \text{ for } x > 1,$$

$$g(x) = x^2 + 2.$$

Find an expression for

(i) $f^{-1}(x)$, [2]

(ii) $gf(x)$, [2]

(iii) $fg(x)$. [2]

(iv) Show that $ff(x) = \dfrac{3x + 2}{x + 2}$ and solve $ff(x) = x$. [4]

(i)	$x = \dfrac{2}{y} + 1 \;\rightarrow\; y = \dfrac{2}{x-1}$	M1	any valid method
	$f^{-1}(x) = \dfrac{2}{x-1}$	A1	
(ii)	$gf(x) = \left(\dfrac{2}{x} + 1\right)^{2} + 2$	B2/1/0	−1 each error
(iii)	$fg(x) = \dfrac{2}{x^2 + 2} + 1$	B2/1/0	−1 each error
(iv)	$ff(x) = \dfrac{2}{\dfrac{2}{x} + 1} + 1 = \dfrac{2x}{x+2} + 1$	M1	correct starting expression
	$= \dfrac{3x+2}{x+2}$	A1	correct algebra to given answer
	$\dfrac{3x+2}{x+2} = x \;\rightarrow\; x^2 - x - 2 = 0$	M1	form and solve 3 term quadratic
	$(x-2)(x+1) = 0$ $x = 2$ only	A1	