SMART EXAM RESOURCES

TOPIC: FUNCTIONS-SET-9

1 **(a)** It is given that $f : x \to 2x^2$ for $x \geqslant 0$,

$g : x \to 2x + 1$ for $x \geqslant 0$.

Each of the expressions in the table can be written as one of the following.

f' f'' g' g'' fg gf f^2 g^2 f^{-1} g^{-1}

Complete the table. The first row has been completed for you. [5]

Expression	Function notation
2	g'
0	
$4x$	
$8x^2 + 8x + 2$	
$4x + 3$	
$\frac{x-1}{2}$	

(b) It is given that $h(x) = (x-1)^2 + 3$ for $x \geqslant a$. The value of a is as small as possible such that h^{-1} exists.

(i) Write down the value of a. [1]

(ii) Write down the range of h. [1]

(iii) Find $h^{-1}(x)$ and state its domain. [3]

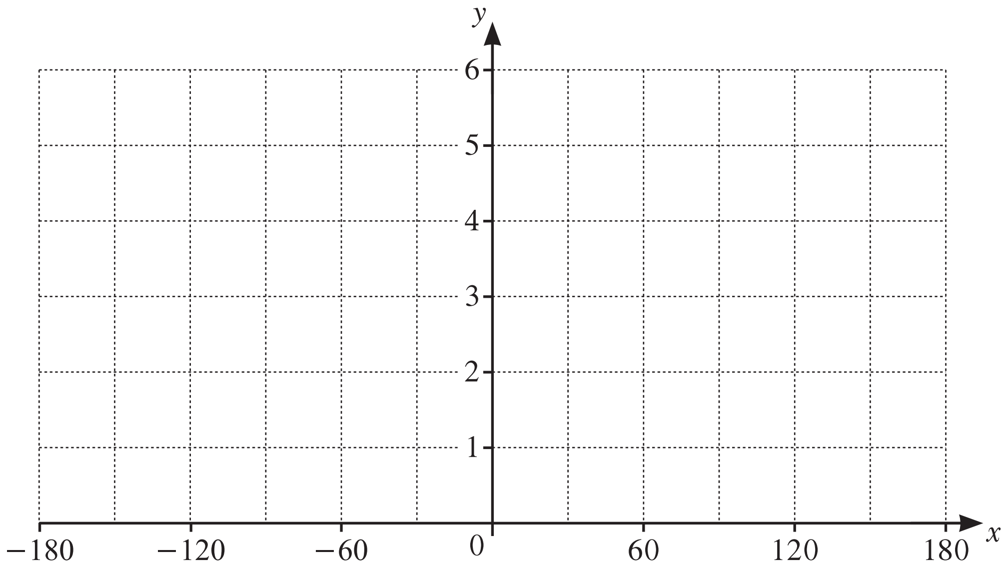
MARK SCHEME:

(a)	<table><tr><td>Expression</td><td>Function notation</td></tr><tr><td>0</td><td>g''</td></tr><tr><td>$4x$</td><td>f'</td></tr><tr><td>$8x^2+8x+2$</td><td>fg</td></tr><tr><td>$4x+3$</td><td>g^2</td></tr><tr><td>$\frac{x-1}{2}$</td><td>g^{-1}</td></tr></table>	**5**	**B1** for each one correct
(b)(i)	$a=1$	**B1**	
(b)(ii)	$h(x)\geqslant 3$	**B1**	
(b)(iii)	$x=(y-1)^2+3$ $y=1+\sqrt{x-3}$	**M1**	For a correct attempt to find the inverse, allow one sign error
	$h^{-1}(x)=1+\sqrt{x-3}$	**A1**	Must be using correct notation
	$x\geqslant 3$	**B1**	Must be using correct notation

2 **(a)** Write down the amplitude of $1+4\cos\left(\frac{x}{3}\right)$. [1]

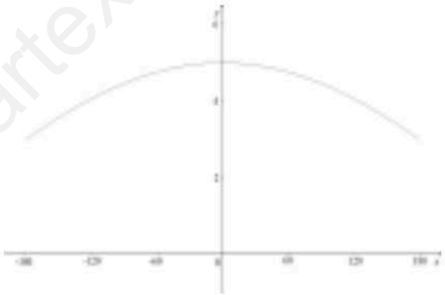
(b) Write down the period of $1+4\cos\left(\frac{x}{3}\right)$. [1]

(c) On the axes below, sketch the graph of $y = 1+4\cos\left(\frac{x}{3}\right)$ for $-180° \leqslant x° \leqslant 180°$.



[3]

MARK SCHEME:

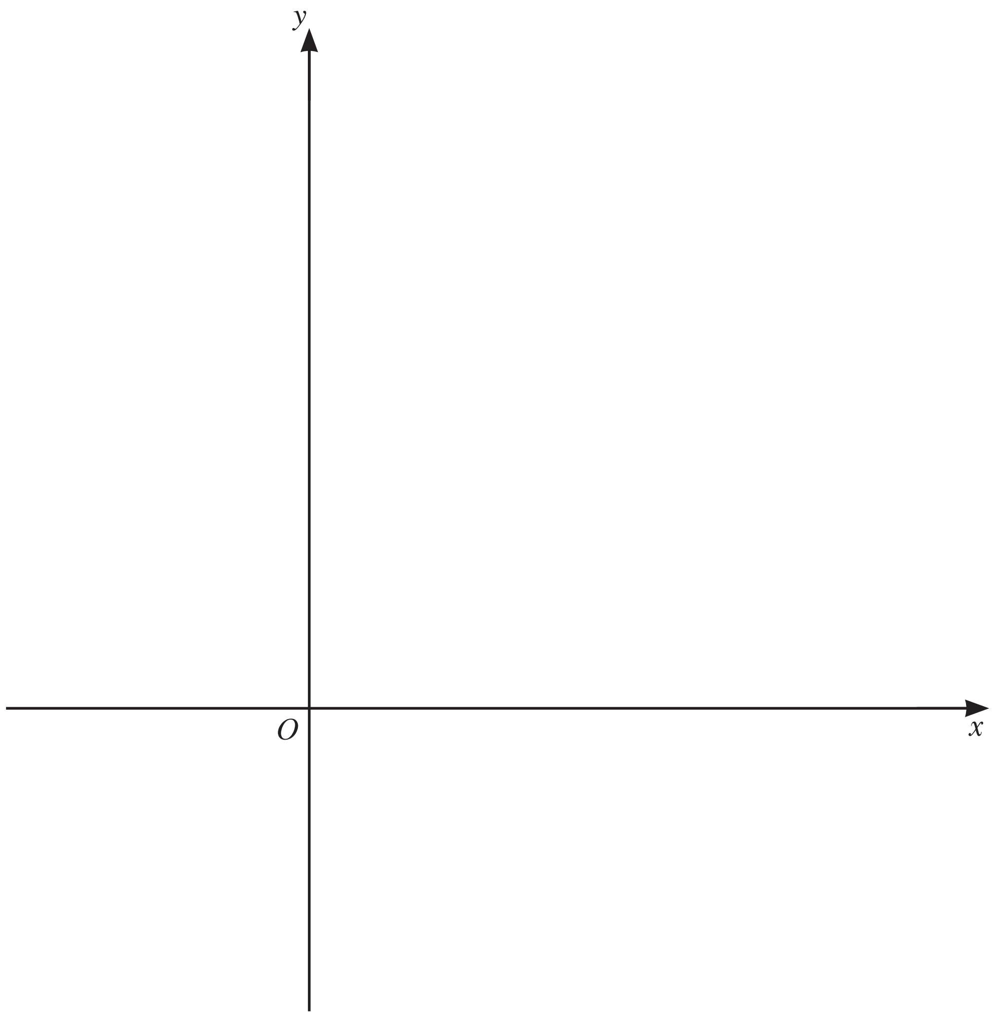
(a)	4	**B1**	
(b)	1080° or 6π	**B1**	
(c)		**3**	**B1** for shape, it must be symmetrical about the y-axis. **B1** for y-intercept of 5 **B1** for $(\pm180^{\circ}, 3)$

3 It is given that $f(x) = 5\ln(2x+3)$ for $x > -\frac{3}{2}$.

(a) Write down the range of f. [1]

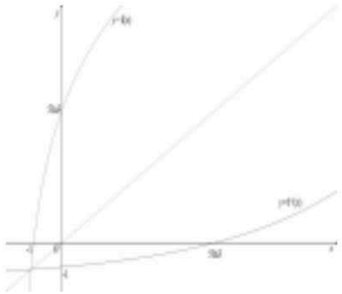
(b) Find f^{-1} and state its domain. [3]

(c) On the axes below, sketch the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. Label each curve and state the intercepts on the coordinate axes.



[5]

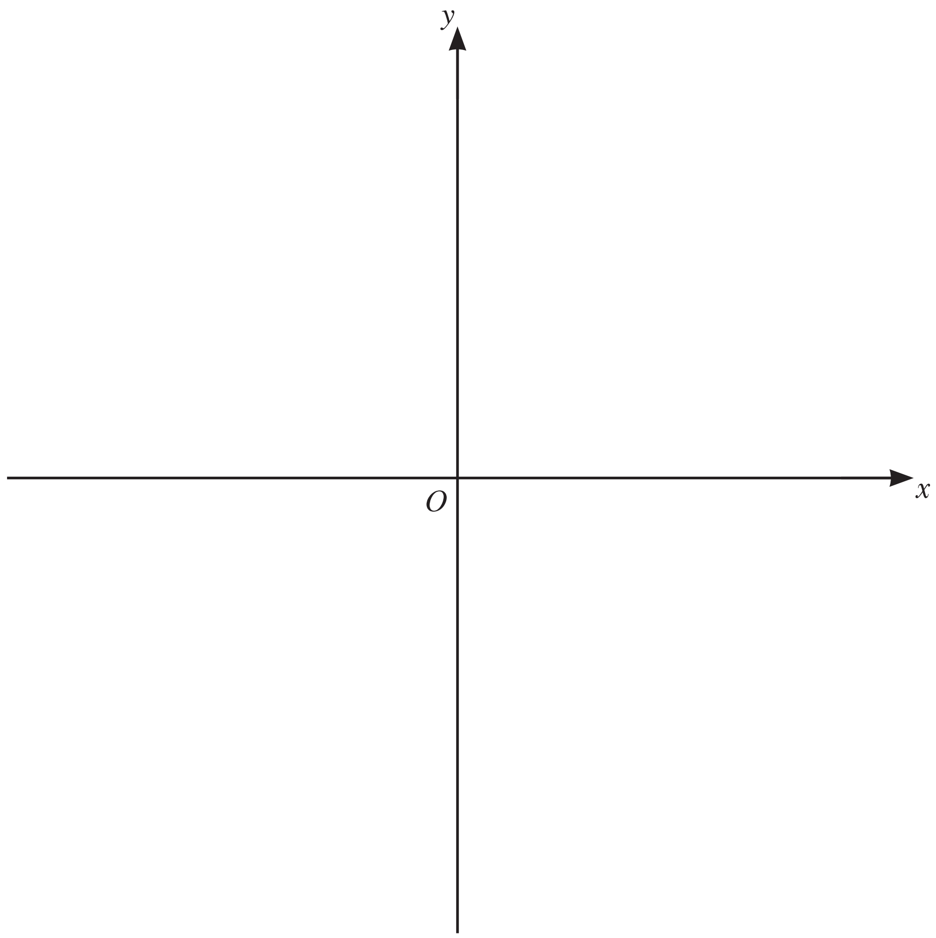
MARK SCHEME:

(a)	$f \in \mathbb{R}$	**B1**	Allow *y* but not *x*
(b)	$x = 5\ln(2y+3)$ $e^{\frac{x}{5}} = 2y+3$	**M1**	For a complete attempt to obtain inverse
	$f^{-1}(x) = \dfrac{e^{\frac{x}{5}} - 3}{2}$	**A1**	Must be using correct notation
	Domain $x \in \mathbb{R}$	**B1**	**FT** on *their* **(a)**. Must be using correct notation
(c)		**5**	**B1** for shape of $y = f(x)$ **B1** for shape of $y = f^{-1}(x)$ **B1** for $5\ln 3$ or 5.5 and -1 on both axes for $y = f(x)$ **B1** for $5\ln 3$ or 5.5 and -1 on both axes for $y = f^{-1}(x)$ **B1** All correct, with apparent symmetry which may be implied be previous 2 B marks or by inclusion of $y = x$, and implied asymptotes, may have one or two points of intersection

4 $$f(x) = x^2 + 2x - 3 \quad \text{for } x \geqslant -1$$

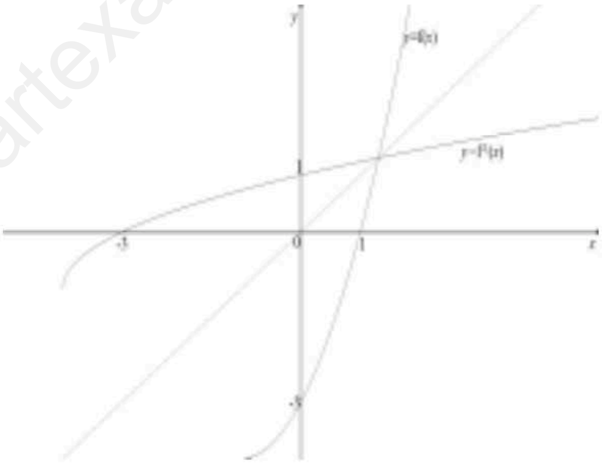
(a) Given that the minimum value of $x^2 + 2x - 3$ occurs when $x = -1$, explain why $f(x)$ has an inverse. [1]

(b) On the axes below, sketch the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. Label each graph and state the intercepts on the coordinate axes.



[4]

MARK SCHEME:

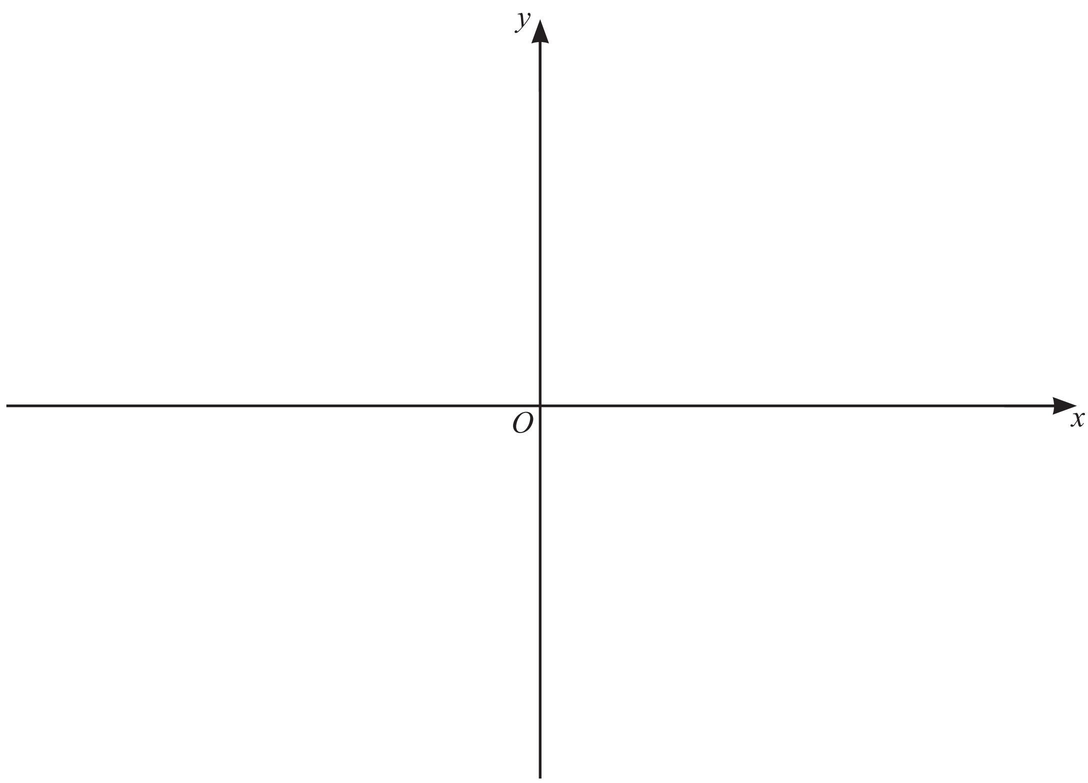
(a)	It is a one-one function because of the given restricted domain or because $x \geqslant -1$	**B1**	
(b)		**4**	**B1** for $y = \mathrm{f}(x)$ for $x > -1$ only **B1** for 1 on x-axis and –3 on y-axis for $y = \mathrm{f}(x)$ **B1** for $y = \mathrm{f}^{-1}(x)$ as a reflection of $y = \mathrm{f}(x)$ in the line $y = x$, maybe implied by intercepts with axes **B1** for 1 on y-axis and –3 on x-axis for $y = \mathrm{f}^{-1}(x)$

5 A function $f(x)$ is such that $f(x) = e^{3x} - 4$, for $x \in \mathbb{R}$.

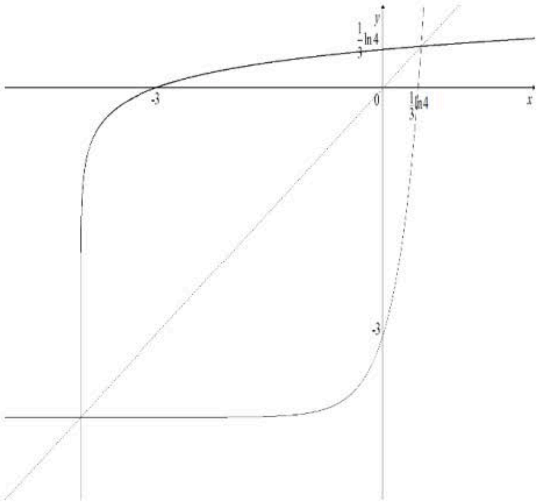
(a) Find the range of f. [1]

(b) Find an expression for $f^{-1}(x)$. [2]

(c) On the axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ stating the exact values of the intercepts with the coordinate axes. [4]



MARK SCHEME:

(a)	$f > -4$	**B1**	Allow $y > -4$ or $-4 < f < \infty$ or $f \in (-4, \infty)$
(b)	$\left[f^{-1}(x) =\right] \frac{1}{3}\ln(x+4)$	**2**	**M1** for a correct method to find the inverse, allow one sign error. Must be in the form of $3x = \ln(y \pm 4)$ or $3y = \ln(x \pm 4)$ **A1** allow $y =$
(c)		**4**	**B1** for $f(x)$ with correct shape in quadrant 1, 3 and 4 and appropriate asymptotic behaviour **B1** for -3 on the y-axis and $\frac{1}{3}\ln 4$ on the x-axis for $f(x)$ must have the correct shape **B1** for $f^{-1}(x)$ with correct shape in quadrant 1, 2 and 3 and appropriate asymptotic behaviour **B1** for -3 on the x-axis and $\frac{1}{3}\ln 4$ on the y-axis for $f^{-1}(x)$ must have correct shape and intersect at least once