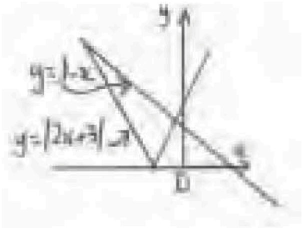
FUNCTIONS-SET-3-QP-MS

1 **(i)** Sketch on the same diagram the graphs of $y = |2x + 3|$ and $y = 1 - x$. [3]

(ii) Find the values of x for which $x + |2x + 3| = 1$. [3]

MARKING SCHEME

(i)  $y = \|2x+3\|$ -ve then +ve slope	B1	Must be 2 parts – ignore -2 to -1
Vertex at (-h,0)	DB1	V shape-Vertex on -ve x-axis + lines
$y = 1 - x$ Line, -ve m, (k,0)	B1 [3]	-ve slope, crosses axes at x,y +ve – allow if only in 1st or 2nd quadrants
(ii) $x + 2x + 3 = 1 \rightarrow x = -\frac{2}{3}$ (-0.65 to -0.70)	B1	From graph, or calculation or guess
$x - (2x+3) = 1 \rightarrow x = -4$ (-3.9 to -4.1)	M1 AI [3]	B2 if correct. M mark for any method. Squares both sides M1 quadratic A1 Answers A1

2 The function f is defined, for $0° \leqslant x \leqslant 360°$, by

$$f(x) = a \sin (bx) + c,$$

where a, b and c are positive integers. Given that the amplitude of f is 2 and the period of f is 120°,

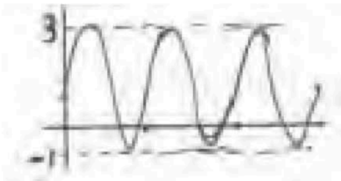
(i) state the value of a and of b. [2]

Given further that the minimum value of f is -1,

(ii) state the value of c, [1]

(iii) sketch the graph of f. [3]

MARKING SCHEME

. x = asin(bx)+c		
(i) a = 2 and b = 3	B1 B1	Wrong way round - no marks. No labels - allow B1 if both correct.
(ii) c = 1	B1	Co
(iii) 3 cycles (0 to 360) -1 to 3	B1 B1	Even if starting incorrectly. Needs to be marked - allow for any trig graph.
 Period 120° + all correct.	DB1	Everything in relatively correct position - needs both B's
	[6]	

3 The function f is defined, for $0° \leqslant x \leqslant 180°$, by

$$f(x) = A + 5\cos Bx,$$

where A and B are constants.

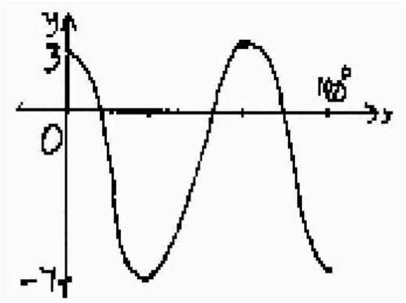
(i) Given that the maximum value of f is 3, state the value of A. [1]

(ii) State the amplitude of f. [1]

(iii) Given that the period of f is 120°, state the value of B. [1]

(iv) Sketch the graph of f. [3]

MARKING SCHEME

$f(x) = A + 5\cos Bx$ **(i)** $A = -2$ **(ii)** Amplitude = 5 **(iii)** $B = 3$ **(iv)** Range 3 to –7	 **B1** **B1** **B1** **B1**	 CAO CAO CAO −3 to 7 implied somewhere – table ok – even if no graph
	**B2,1** **[6]**	Needs 1½ oscillations – over-rides rest. √ on 3 and –7 Start at max – finishes at second min. Curves – but be tolerant

4 Given that each of the following functions is defined for the domain $-2 \leqslant x \leqslant 3$, find the range of

(i) $f : x \mapsto 2 - 3x$, [1]

(ii) $g : x \mapsto |2 - 3x|$, [2]

(iii) $h : x \mapsto 2 - |3x|$. [2]

State which of the functions f, g and h has an inverse. [2]

MARKING SCHEME

(i) $-7 \le f(x) \le 8$ **(i)** $0 \le g(x) \le 8$ **(ii)** $-7 \le h(x) \le 2$ f yes g no h no	**B1** **B1 B1** **B1 B1** **B2,1** **[7]**	CAO Allow < for ≤ CAO As above CAO As above Loses one for each wrong decision. (answer f on its own – allow B2)

5 The function f is defined, for $0° \leqslant x \leqslant 180°$, by

$$f(x) = 3\cos 4x - 1.$$

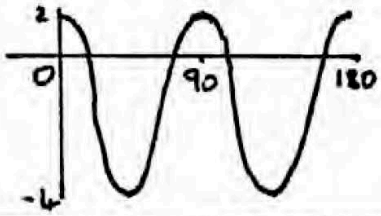
(i) Solve the equation $f(x) = 0$. [3]

(ii) State the amplitude of f. [1]

(iii) State the period of f. [1]

(iv) State the maximum and minimum values of f. [2]

(v) Sketch the graph of $y = f(x)$. [3]

$f(x) = 3\cos 4x - 1$.		
(i) $\cos 4x = ⅓$ (base angle = 70.53)	M1	cos4x subject then ÷ by 4
4x=70.53 or 289.47 or 430.53 or 649.47		
	A1	One pair correct.
x = 17.6° or 72.4° or 107.6° or 162.4°	A1√	Other pair correct to first answers.
	[3]	
(ii) amplitude = 3	B1	Co
(iii) period = 90^0 or ½π	B1	Co
(iv) maximum value = 3-1 = 2	B1	Co
minimum value = -3 –1 = - 4	B1	Co
	[4]	
(v) 	B1 B1√ B1 [3]	• 2 complete cycles • Max "amp –1" Min "-amp-1" • Starts and finishes correctly

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(a) Functions f and g are defined, for $x \in \mathbb{R}$, by

$$f(x) = 3 - x,$$
$$g(x) = \frac{x}{x + 2}, \quad \text{where } x \neq -2.$$

(i) Find fg(x). [2]

(ii) Hence find the value of x for which fg(x) = 10. [2]

(b) A function h is defined, for $x \in \mathbb{R}$, by $h(x) = 4 + \ln x$, where $x > 1$.

(i) Find the range of h. [1]

(ii) Find the value of $h^{-1}(9)$. [2]

(iii) On the same axes, sketch the graphs of $y = h(x)$ and $y = h^{-1}(x)$. [3]

MARKING SCHEME

(a) **(i)** $\text{fg}(x) = \text{f}\left(\frac{x}{x+2}\right)$	M1	M1 for order
$= 3 - \frac{x}{x+2}$	A1 [2]	
(ii) $3 - \frac{x}{x+2} = 10$	DM1	DM1 for dealing with fractions sensibly
leading to $x = -1.75$	A1 [2]	
(b) **(i)** $\text{h}(x) > 4$	B1 [1]	
(ii) $\text{h}^{-1}(x) = \text{e}^{x-4}$	M1	M1 for attempting to obtain inverse function
$\text{h}^{-1}(9) = \text{e}^5$ (≈ 148)	A1 [2]	
or $4 + \ln x = 9$,		
leading to $x = \text{e}^5$		
(iii) correct graphs	B1	B1 for each curve
	B1	
	B1 [3]	B1 for idea of symmetry

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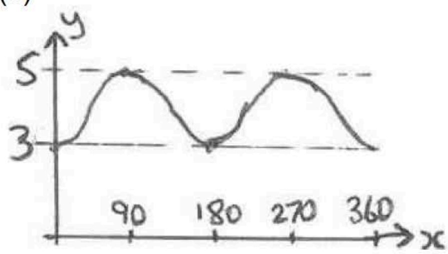
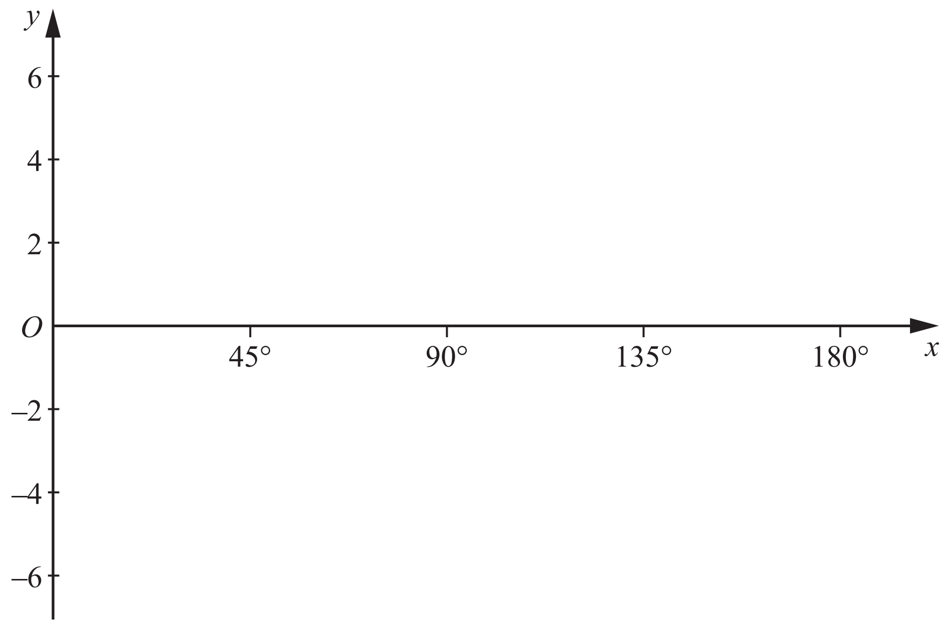
The function f is defined, for $0° \leqslant x \leqslant 360°$, by $f(x) = 4 - \cos 2x$.

(i) State the amplitude and period of f. [2]

(ii) Sketch the graph of f, stating the coordinates of the maximum points. [4]

. f(x) = 4 – cos2x		
(i) amplitude = $\pm$1. Period = 180° or π	B1B1	Independent of graph. Do not allow "4 to 5".
(ii) 	B2,1	Must be two complete cycles. 0/2 if not. Needs 3 to 5 marked or implied. Needs to start and finish at minimum. Needs curve not lines.
Max (90°, 5) and (270°, 5)	B1B1 [6]	Independent of graph (90, 270 gets B1). Allow radians or degrees.

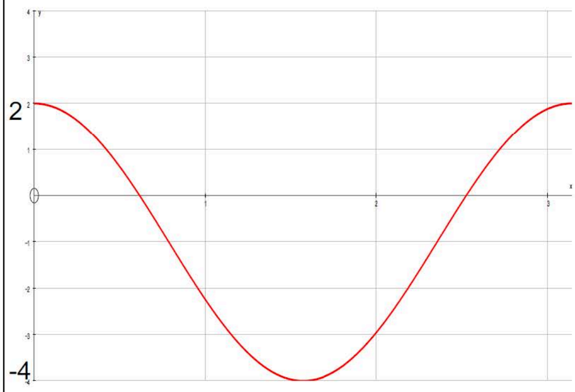
8 **(a)** On the axes below, sketch the curve $y = 3\cos 2x - 1$ for $0° \leqslant x \leqslant 180°$. [3]



(b) **(i)** State the amplitude of $1 - 4\sin 2x$. [1]

(ii) State the period of $5\tan 3x + 1$. [1]

MARKING SCHEME

(a)		B1 B1 B1	for correct shape for max value of 2, starting at (0, 2) and finishing at (180°, 2) for min value of –4
(b) (i)	4	B1	must be positive
(ii)	60° or $\frac{\pi}{3}$ or 1.05 rad	B1	