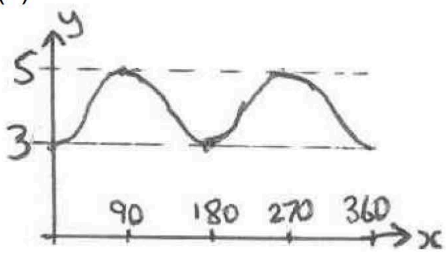
FUNCTIONS-SET-1-QP-MS

1 The function f is defined, for $0° \leqslant x \leqslant 360°$, by $f(x) = 4 - \cos 2x$.

(i) State the amplitude and period of f. [2]

(ii) Sketch the graph of f, stating the coordinates of the maximum points. [4]

MARKING SCHEME:

f(x) = 4 – cos2x		
(i) amplitude = ± 1. Period = 180^o or π	B1B1	Independent of graph. Do not allow "4 to 5".
(ii) 	B2,1	Must be two complete cycles. 0/2 if not. Needs 3 to 5 marked or implied. Needs to start and finish at minimum. Needs curve not lines.
Max (90^o, 5) and (270^o, 5)	B1B1 [6]	Independent of graph (90, 270 gets B1). Allow radians or degrees.

2 Given that each of the following functions is defined for the domain $-2 \leqslant x \leqslant 3$, find the range of

(i) $f : x \mapsto 2 - 3x$, [1]

(ii) $g : x \mapsto |2 - 3x|$, [2]

(iii) $h : x \mapsto 2 - |3x|$. [2]

State which of the functions f, g and h has an inverse. [2]

MARKING SCHEME:

(i) $-7 \le f(x) \le 8$	**B1**	CAO Allow < for ≤
(i) $0 \le g(x) \le 8$	**B1 B1**	CAO As above
(ii) $-7 \le h(x) \le 2$	**B1 B1**	CAO As above
f yes g no h no	**B2,1** **[7]**	Loses one for each wrong decision. (answer f on its own – allow B2)

3 **(a)** The function f is such that $f(x) = 2x^2 - 8x + 5$.

(i) Show that $f(x) = 2(x + a)^2 + b$, where a and b are to be found. [2]

(ii) Hence, or otherwise, write down a suitable domain for f so that f^{-1} exists. [1]

(b) The functions g and h are defined respectively by

$$g(x) = x^2 + 4, \quad x \geqslant 0, \quad h(x) = 4x - 25, x \geqslant 0.$$

(i) Write down the range of g and of h^{-1}. [2]

(ii) On the axes below, sketch the graphs of $y = g(x)$ and $y = g^{-1}(x)$, showing the coordinates of any points where the curves meet the coordinate axes. [3]

(iii) Find the value of x for which $gh(x) = 85$. [4]

MARKING SCHEME:

(i) $2(x-2)^2-3$	B1B1	B1 for –2, B1 for –3
(ii) $x \geq 2$ or equivalent	√B1	√ on their '–2'
(b) (i) $g(x) \geq 4$, $h^{-1}(x) \geq 0$	B1B1	B1 for each
(ii) Correct sketch	B1 B1 B1	B1 for g(x) B1 for $g^{-1}(x)$ B1 for idea of symmetry
(iii) $g(4x-25)=85$	M1	M1 for correct order
$(4x-25)^2+4=85$	DM1	DM1 for attempt to solve
$x=\frac{17}{2}$, $x=4$	A1	A1 for both
Discarding $x=4$	B1 [12]	B1 for discarding $x=4$

4 It is given that $f(x) = 3e^{2x}$ for $x \geqslant 0$,
$g(x) = (x+2)^2 + 5$ for $x \geqslant 0$.

(i) Write down the range of f and of g. [2]

(ii) Find g^{-1}, stating its domain. [3]

(iii) Find the exact solution of $gf(x) = 41$. [4]

(iv) Evaluate $f'(\ln 4)$. [2]

MARKING SCHEME:

(i)	Range for f: $y \geq 3$ Range for g: $y \geq 9$	**B1** **B1**	
(ii)	$x = -2 + \sqrt{y-5}$	**M1**	attempt to obtain the inverse function
	$g^{-1}(x) = -2 + \sqrt{x-5}$ Domain of g^{-1}: $x \geq 9$	**A1** **B1**	Must be correct form for domain
	Alternative method: $y^2 + 4y + 9 - x = 0$	**M1**	attempt to use quadratic formula and find inverse
	$y = \dfrac{-4 + \sqrt{16 - 4(9-x)}}{2}$	**A1**	must have + not ±
(iii)	Need $g\left(3e^{2x}\right)$	**M1**	correct order
	$\left(3e^{2x} + 2\right)^2 + 5 = 41$ or $9e^{4x} + 12e^{2x} - 32 = 0$ $\left(3e^{2x} - 4\right)\left(3e^{2x} + 8\right) = 0$	**DM1**	correct attempt to solve the equation
	leading to $3e^{2x} + 2 = \pm 6$ so $x = \frac{1}{2}\ln\frac{4}{3}$	**M1**	dealing with the exponential correctly in order to reach a solution for x
	or $e^{2x} = \frac{4}{3}$ so $x = \frac{1}{2}\ln\frac{4}{3}$	**A1**	Allow equivalent logarithmic forms
	Alternative method: Using $f(x) = g^{-1}(41)$, $g^{-1}(41) = 4$	**M1**	correct use of g^{-1}
	leading to $3e^{2x} = 4$, so $x = \frac{1}{2}\ln\frac{4}{3}$	**DM1**	dealing with $g^{-1}(41)$ to obtain an equation in terms of e^{2x}
		M1 **A1**	dealing with the exponential correctly in order to reach a solution for x Allow equivalent logarithmic forms
(iv)	$g'(x) = 6e^{2x}$ $g'(\ln 4) = 96$	**B1** **B1**	B1 for each

5 It is given that $f(x) = 3e^{2x}$ for $x \geqslant 0$,
$g(x) = (x+2)^2 + 5$ for $x \geqslant 0$.

(i) Write down the range of f and of g. [2]

(ii) Find g^{-1}, stating its domain. [3]

(iii) Find the exact solution of $gf(x) = 41$. [4]

(iv) Evaluate $f'(\ln 4)$. [2]

MARKING SCHEME:

(i)	Range for f: $y \geq 3$ Range for g: $y \geq 9$	**B1** **B1**	
(ii)	$x = -2 + \sqrt{y-5}$	**M1**	attempt to obtain the inverse function
	$g^{-1}(x) = -2 + \sqrt{x-5}$ Domain of g^{-1}: $x \geq 9$	**A1** **B1**	Must be correct form for domain
	Alternative method: $y^2 + 4y + 9 - x = 0$	**M1**	attempt to use quadratic formula and find inverse
	$y = \dfrac{-4 + \sqrt{16 - 4(9-x)}}{2}$	**A1**	must have + not ±
(iii)	Need $g(3e^{2x})$	**M1**	correct order
	$(3e^{2x} + 2)^2 + 5 = 41$ or $9e^{4x} + 12e^{2x} - 32 = 0$ $(3e^{2x} - 4)(3e^{2x} + 8) = 0$	**DM1**	correct attempt to solve the equation
	leading to $3e^{2x} + 2 = \pm 6$ so $x = \frac{1}{2}\ln\frac{4}{3}$	**M1**	dealing with the exponential correctly in order to reach a solution for x
	or $e^{2x} = \frac{4}{3}$ so $x = \frac{1}{2}\ln\frac{4}{3}$	**A1**	Allow equivalent logarithmic forms
	Alternative method: Using $f(x) = g^{-1}(41)$, $g^{-1}(41) = 4$	**M1**	correct use of g^{-1}
	leading to $3e^{2x} = 4$, so $x = \frac{1}{2}\ln\frac{4}{3}$	**DM1**	dealing with $g^{-1}(41)$ to obtain an equation in terms of e^{2x}
		M1 **A1**	dealing with the exponential correctly in order to reach a solution for x Allow equivalent logarithmic forms
(iv)	$g'(x) = 6e^{2x}$ $g'(\ln 4) = 96$	**B1** **B1**	B1 for each

6 The function f is defined for the domain $-3 \leqslant x \leqslant 3$ by

$$f(x) = 9\left(x - \frac{1}{3}\right)^2 - 11.$$

(i) Find the range of f. [3]

(ii) State the coordinates and nature of the turning point of

(a) the curve $y = f(x)$,

(b) the curve $y = |f(x)|$.

[4]

MARKING SCHEME:

$f(x)=9(x-\frac{1}{3})^2-11$		
Minimum at $x=\frac{1}{3}$	M1 A1	Correct method for x co-ord of min.pt.
(i) Range is -11 to 89.	B1 B1	B1 for each value. ≥ 89 gets B0.
(ii) (a) ($\frac{1}{3}$, -11) Minimum.	B1	For "Minimum" – ignore any working.
(b) ($\frac{1}{3}$, 11) Maximum	B1√B1√ [7]	Correct follow through from his coordinates and nature of stationary point.

7 **(i)** Sketch the graph of $y = |3x - 5|$, for $-2 \leqslant x \leqslant 3$, showing the coordinates of the points where the graph meets the axes. [3]

(ii) On the same diagram, sketch the graph of $y = 8x$. [1]

(iii) Solve the equation $8x = |3x - 5|$. [3]

MARKING SCHEME:

(i) Graph of modulus function	B1 B1 B1 [3]	B1 for shape B1 for 5 marked on y axis B1 for $\frac{5}{3}$ marked on x axis
(ii) Straight line graph	B1 [1]	B1 for straight line with greater gradient
(iii) $8x = \pm(3x - 5)$ leading to $x = \frac{5}{11}$ or 0.455 **only**	M1 M1, A1 [3]	M1 for attempt to deal with modulus M1 for solution

8 **(a)** A function f is defined, for $x \in \mathbb{R}$, by

$$f(x) = x^2 + 4x - 6.$$

(i) Find the least value of $f(x)$ and the value of x for which it occurs. [2]

(ii) Hence write down a suitable domain for $f(x)$ in order that $f^{-1}(x)$ exists. [1]

(b) Functions g and h are defined, for $x \in \mathbb{R}$, by

$$g(x) = \frac{x}{2} - 1,$$

$$h(x) = x^2 - x.$$

(i) Find $g^{-1}(x)$. [2]

(ii) Solve $gh(x) = g^{-1}(x)$. [3]

MARKING SCHEME:

(a) (i) $f_{min} = -10$, occurs when $x = -2$	B1 B1 [2]	
(ii) e.g. $x \geqslant -2$	B1 [1]	Allow any suitable domain that makes f a 1:1 function
(b) (i) $x = \left(\frac{y}{2} - 1\right)$, leading to $g^{-1}(x) = 2(x + 1)$	M1 A1 [2]	M1 for a valid method of finding the inverse function
(ii) $\frac{x^2 - x}{2} - 1 = 2(x + 1)$ leading to $x^2 - 5x - 6 = 0$ solution $x = 6$ and -1	M1 DM1 A1 [3]	M1 for correct order DM1 for solution of quadratic