FUNCTIONS

2.10

Answer only **one** of the following two alternatives.

EITHER

The functions f and g are defined, for $x > 1$, by

$$f(x) = (x + 1)^2 - 4,$$

$$g(x) = \frac{3x + 5}{x - 1}.$$

Find

(i) fg(9), [2]

(ii) expressions for $f^{-1}(x)$ and $g^{-1}(x)$, [4]

(iii) the value of x for which $g(x) = g^{-1}(x)$. [4]

OR

A particle moves in a straight line so that, at time t s after passing a fixed point O, its velocity is $v\,\text{ms}^{-1}$, where

$$v = 6t + 4\cos 2t.$$

Find

(i) the velocity of the particle at the instant it passes O, [1]

(ii) the acceleration of the particle when $t = 5$, [4]

(iii) the greatest value of the acceleration, [1]

(iv) the distance travelled in the fifth second. [4]

12E

(i) fg(9) = f(4) evaluated or $fg(x) = \left(\frac{3x+5}{x-1}+1\right)^2 - 4$	M1
21	A1
(ii) Method for $f^{-1}(x)$	M1
$f^{-1}(x) = \sqrt{x+4} - 1$	A1
Put $y = \frac{3x+5}{x-1}$ and rearrange	M1
$g^{-1}(x) = \frac{x+5}{x-3}$	A1
(iii) Rearrange two of $\frac{3x+5}{x-1} = \frac{x+5}{x-3} = x$ to quadratic equation	M1
$2(x^2 - 4x - 5) = 0$	A1
Solve 3 term quadratic	M1
5 only	A1
	[10]

12O

(i) 4	B1
(ii) Differentiate v to find an expression for a	M1
$6 - 8 \sin 2t$	A1
Substitute $t = 5$	DM1
10.3 to 10.4	A1
(iii) 14	B1
(iv) Integrate v to find an expression for s	M1
$s = 3t^2 + 2 \sin 2t$	A1
Use limits 4 and 5	DM1
23.9	A1
	[10]

2.11

(a) The functions f and g are defined, for $x \in \mathbb{R}$, by

$$f : x \mapsto 2x + 3,$$
$$g : x \mapsto x^2 - 1.$$

Find fg(4). [2]

(b) The functions h and k are defined, for $x > 0$, by

$$h : x \mapsto x + 4,$$
$$k : x \mapsto \sqrt{x}\ .$$

Express each of the following in terms of h and k.

(i) $x \mapsto \sqrt{x + 4}$ [1]

(ii) $x \mapsto x + 8$ [1]

(iii) $x \mapsto x^2 - 4$ [2]

(a)	f(15) evaluated or $fg(x) = 2(x^2 - 1) + 3$	M1
	33	A1
(b)	**(i)** kh	B1
	(ii) h^2 or hh	B1
	(iii) $h^{-1}k^{-1}$ or $(kh)^{-1}$	B2
		[6]

Answer only **one** of the following alternatives.

EITHER

(i) Express $4x^2 + 32x + 55$ in the form $(ax + b)^2 + c$, where a, b and c are constants and a is positive. [3]

The functions f and g are defined by

$$\mathrm{f} : x \mapsto 4x^2 + 32x + 55 \text{ for } x > -4,$$

$$\mathrm{g} : x \mapsto \frac{1}{x} \text{ for } x > 0.$$

(ii) Find $\mathrm{f}^{-1}(x)$. [3]

(iii) Solve the equation $\mathrm{fg}(x) = 135$. [4]

OR

The functions h and k are defined by

$$\mathrm{h} : x \mapsto \sqrt{2x - 7} \text{ for } x \geqslant c,$$

$$\mathrm{k} : x \mapsto \frac{3x - 4}{x - 2} \text{ for } x > 2.$$

(i) State the least possible value of c. [1]

(ii) Find $h^{-1}(x)$. [2]

(iii) Solve the equation $\mathrm{k}(x) = x$. [3]

(iv) Find an expression for the function k^2, in the form $\mathrm{k}^2 : x \mapsto a + \frac{b}{x}$ where a and b are constants. [4]

Start your answer to Question 12 here.

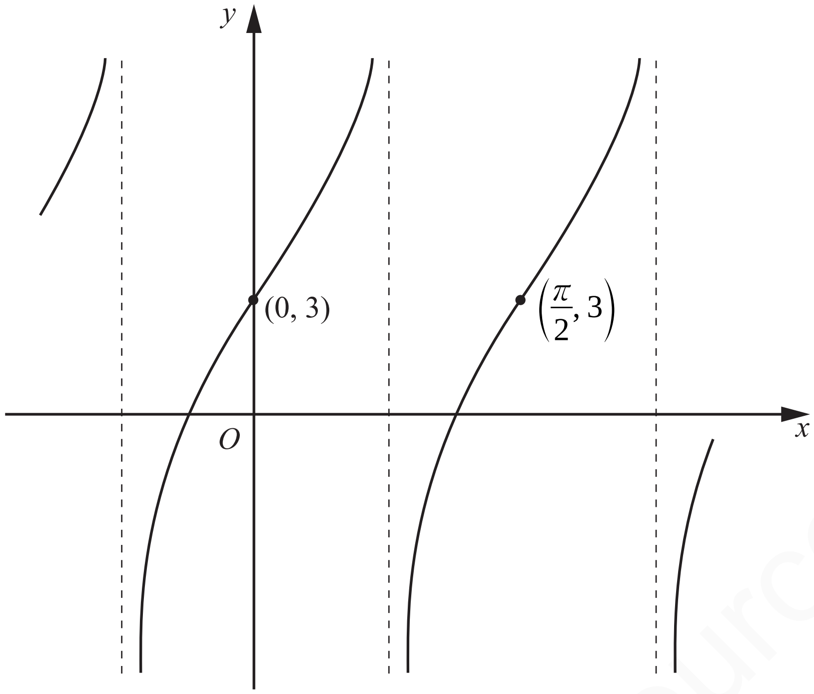
Indicate which question you are answering.

EITHER	
OR	

Continue your answer here if necessary.

(i) $(2x+8)^2-9$ or $a=2, b=8, c=-9$	B1B1B1 [3]	B1 for each correct value
(ii) $f^{-1}(x)=\dfrac{\sqrt{(x+9)}-8}{2}$ oe	M1	inverse of form $\dfrac{\sqrt{(x\pm c)}\pm b}{a}$
	A2,1,0√ [3]	3, 1 – 2, 0 correct values, ft their a, b and c
(iii) $\left(\dfrac{2}{x}+8\right)^2-9=135$ or $\dfrac{4}{x^2}+\dfrac{32}{x}+55=135$	M1	apply fg (not gf) or replace x by $\dfrac{1}{x}$
$\dfrac{2}{x}+8=12(\text{or } -12)$ or $80x^2-32x-4=0$	A1	correct equation
	M1	valid method for solving their equation
$x=0.5$ oe, only	A1 [4]	correct answer
(i) 3.5	B1 [1]	correct answer
(ii) $y^2+7=2x$	M1	attempt at inverse, involving squaring
$h^{-1}(x)=\dfrac{x^2+7}{2}$	A1 [2]	correct inverse
(iii) $\dfrac{3x-4}{x-2}=x,\ x^2-5x+4=0$	M1	equate $k(x)$ with x and obtain quadratic equation
$(x-4)(x-1)$	M1	solve three term quadratic
$x=4$ only	A1 [3]	correct answer
(iv) $\dfrac{3\left(\dfrac{3x-4}{x-2}\right)-4}{\left(\dfrac{3x-4}{x-2}\right)-2}$	M1	substitute to obtain expression for k^2
	A1	correct unsimplified expression
$\dfrac{3(3x-4)-4(x-2)}{3x-4-2(x-2)}$	M1	multiply numerator and denominator by $(x-2)$, oe
$5-\dfrac{4}{x}$	A1 [4]	correct answer

2.12



(a) **(i)** The diagram shows the graph of $y = A + C\tan(Bx)$ passing through the points (0, 3) and $\left(\frac{\pi}{2}, 3\right)$. Find the value of A and of B. [2]

(ii) Given that the point $\left(\frac{\pi}{8}, 7\right)$ also lies on the graph, find the value of C. [1]

(b) Given that $f(x) = 8 - 5\cos 3x$, state the period and the amplitude of f. [2]

period .... amplitude ....

(a) **(i)**	$A = 3$, $B = 2$	B1, B1	
(ii)	$C = 4$	B1	
(b)	120 or $\frac{2\pi}{3}$	B1	
	5	B1	