FUNCTIONS

2.5

A one-one function f is defined by $f(x) = (x - 1)^2 - 5$ for $x \geqslant k$.

For Examiner's Use

(i) State the least value that k can take. [1]

For this least value of k

(ii) write down the range of f, [1]

(iii) find $f^{-1}(x)$, [2]

(i)	1	**B1**	Not a range for *k*, but condone $x = 1$ and x 1
(ii)	f –5	**B1**	Not *x*, but condone *y*
(iii)	Method of inverse	**M1**	Do not reward poor algebra but allow slips
	$1+\sqrt{x+5}$	**A1**	Must be f^{-1} = ...or y = ...

2.6

The function f is such that $\mathrm{f}(x) = 2 + \quad x\sqrt{\ 3}$ for $4 \leqslant x \leqslant 28$.

(i) Find the range of f. [2]

(ii) Find $\mathrm{f}^2(12)$. [2]

(iii) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

The function g is defined by $\mathrm{g}(x) = \dfrac{120}{x}$ for $x \geqslant 0$.

(iv) Find the value of x for which $\mathrm{gf}(x) = 20$. [3]

(i)	$3 < f < 7$	**B1,B1**	If **B0** then **SC1** for $3 \le f \le 7$
(ii)	$f(12) = 5$	**B1**	$f^2(x)$ $\sqrt{\left(\sqrt{(x-3)} + 2 - 3\right)} + 2$ earns **B1**
	$(f(5) =)\ 2 + \sqrt{2}$	**B1**	
(iii)	Clear indication of method	**M1**	
	$f^{-1}(x) = (x-2)^2 + 3$	**A1**	condone $y = (x-2)^2 + 3$
(iv)	$gf(x) = \dfrac{120}{\sqrt{(x-3)} + 2}$	**B1**	
	Attempt to solve *their* $gf(x) = 20$	**M1**	
	$x = 19$	**A1**	

2.7

Answer only **one** of the following two alternatives.

EITHER

Functions f and g are defined for $x \in \mathbb{R}$ by

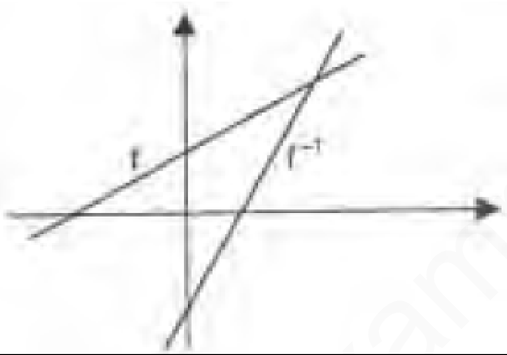
$$f : x \mapsto 3x - 2, \quad x \neq \tfrac{4}{3},$$
$$g : x \mapsto \frac{4}{2 - x}, \quad x \neq 2.$$

(i) Solve the equation $gf(x) = 2$. [3]

(ii) Determine the number of real roots of the equation $f(x) = g(x)$. [2]

(iii) Express f^{-1} and g^{-1} in terms of x. [3]

(iv) Sketch, on a single diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, stating the coordinates of the point of intersection of the two graphs. [3]

12 EITHER [11]	(i)	gf(x) = $\dfrac{4}{2-(3x-2)}$	B1
		Solve $\dfrac{4}{4-3x}=2$ [or solve fg(x) = $3\left(\dfrac{4}{2-x}\right)-2=2$]	M1
		$\Rightarrow x = 2/3$	A1
	(ii)	$f(x) = g(x) \Rightarrow 3x-2 = \dfrac{4}{2-x} \Rightarrow 3x^2-8x+8=0$	
		Discriminant = 64 – 96 < 0 $\Rightarrow$ No real roots	M1 A1
	(iii)	$f^{-1}: x \mapsto (x+2) \div 3$	B1
		$y = 4/(2-x) \Rightarrow x = 2 - 4/y \Rightarrow g^{-1}: x \mapsto 2-4/x$	M1 A1
	(iv)		B1 B1
		Lines intersect at (1, 1)	B1

2.8

The function f is defined by f: $x \mapsto |x^2 - 8x + 7|$ for the domain $3 \leqslant x \leqslant 8$.

(i) By first considering the stationary value of the function $x \mapsto x^2 - 8x + 7$, show that the graph of $y = \mathrm{f}(x)$ has a stationary point at $x = 4$ and determine the nature of this stationary point. [4]

(ii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(iii) Find the range of f. [2]

The function g is defined by g: $x \mapsto |x^2 - 8x + 7|$ for the domain $3 \leqslant x \leqslant k$.

(iv) Determine the largest value of k for which g^{-1} exists. [1]

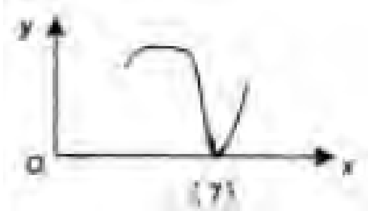
[9] **(i)** Let $y = x^2 - 8x + 7$ $dy/dx = 2x - 8 = 0$ at $x = 4$ — M1

$d^2y/dx^2 = 2$ $\therefore$ min at $x = 4$ — A1

OR via completing the square: $y = (x - 4)^2 - 9 \Rightarrow$ min -9 at $x = 4$

$\therefore$ f(x) has maximum at $x = 4$, corroborated by argument re reflection of -9 or by graph — B2, 1, 0

(ii)



B2, 1, 0

Judge by shape, unless values clearly incorrect.

Ignore curve outside domain.

Cusp needed at x-axis.

Accept straight line for right-hand arm, but curvature, if shown, must be correct.

(iii) $0 \le f(x) \le 9$ [condone <] — B1 B1

(iv) $k = 4$ — B1

2.9

Solve the equation $|2x + 10| = 7$. [3]

-1.5	B1
Solve $2x + 10 = -7$ or $(2x + 10)^2 = 49$	M1
-8.5	A1
	[3]