# FUNCTIONS-SET-5-QP-MS

**1** **(a)** $f(x) = 4\ln(2x-1)$

**(i)** Write down the largest possible domain for the function f. [1]

**(ii)** Find $f^{-1}(x)$ and its domain. [3]

**(b)**

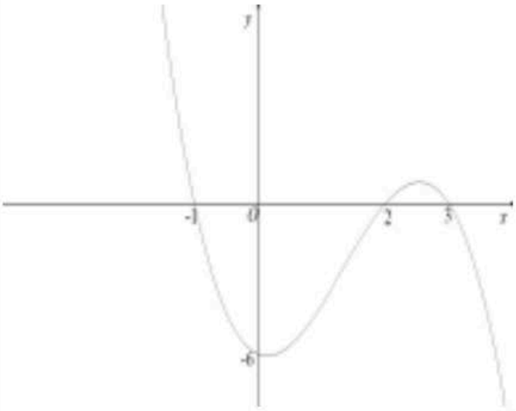
$g(x) = x+5$ for $x \in \mathbb{R}$

$h(x) = \sqrt{2x-3}$ for $x \geqslant \frac{3}{2}$

Solve $gh(x) = 7$. [3]

www.smartexamresources.com

## MARKING SCHEME

| | | | |
|---|---|---|---|
| (a) |  | **3** | **B1** for a well-drawn cubic graph in correct orientation<br>Both arms extending beyond $x$-axis<br>Maximum above $x$-axis<br>**B1** for $x$-intercepts<br>**B1** for $y$-intercept |
| 1(b) | $x < -1$ | **B1** | Dep on a cubic curve in the correct orientation and –1 correct on $x$-axis |
| | $2 < x < 3$ or $3 > x > 2$ | **B1** | Dep on a cubic curve in the correct orientation and 2 and 3 correct on $x$-axis |

# 2

**(a)** It is given that $x(\ )$ f e for $x \in \mathbb{R}$. $+$ $^{4}3$ $5$ $x$

**(i)** State the range of f. [1]

**(ii)** Find $\mathrm{f}^{-1}$ and state its domain. [4]

**(b)** It is given that $\mathrm{g}(x) = x^2 + 5$ and $\mathrm{h}(x) = \ln x$ for $x > 0$. Solve $\mathrm{hg}(x) = 2$. [3]

## MARKING SCHEME

| | | | |
|---|---|---|---|
| (a)(i) | $f > 5$, $f(x) > 5$ | **B1** | |
| (a)(ii) | $\frac{y-5}{3} = e^{-4x}$ or $\frac{x-5}{3} = e^{-4y}$ | **B1** | |
| | $-4x = \ln\left(\frac{y-5}{3}\right)$ or $-4y = \ln\left(\frac{x-5}{3}\right)$ | **B1** | |
| | leading to $f^{-1}(x) = -\frac{1}{4}\ln\left(\frac{x-5}{3}\right)$<br>or $f^{-1}(x) = \frac{1}{4}\ln\left(\frac{3}{x-5}\right)$<br>or $f^{-1}(x) = \frac{1}{4}(\ln 3 - \ln(x-5))$<br>or $f^{-1}(x) = -\frac{1}{4}(\ln(x-5) - \ln 3)$ | **B1** | |
| | Domain $x > 5$ | **B1** | |
| (b) | $\ln(x^2+5) = 2$ | **B1** | |
| | $x^2 + 5 = e^2$ | **B1** | |
| | $x = 1.55$ or better or $\sqrt{e^2 - 5}$ | **B1** | |

www.smartexamresources.com

**3** It is given that $\mathrm{f}(x) = 5\mathrm{e}^{x} - 1$ for $x \in \mathbb{R}$.

**(i)** Write down the range of f. [1]

**(ii)** Find $\mathrm{f}^{-1}$ and state its domain. [3]

It is given also that $\mathrm{g}(x) = x^2 + 4$ for $x \in \mathbb{R}$.

**(iii)** Find the value of fg(1). [2]

www.smartexamresources.com

**(iv)** Find the exact solutions of $g^2(x) = 40$. [3]

www.smartexamresources.com

# MARKING SCHEME

| | | | |
|---|---|---|---|
| (i) | $f > -1$ | B1 | or $f(x) > -1$, $y > -1$, $(-1, \infty)$, $\{y : y > -1\}$ |
| (ii) | $e^y = \frac{x+1}{5}$ oe | M1 | a complete valid method to obtain the inverse function |
| | $y = \ln\left(\frac{x+1}{5}\right)$ or $f^{-1}(x) = \ln\left(\frac{x+1}{5}\right)$ oe | A1 | |
| | Domain $x > -1$ or $(-1, \infty)$ | B1 | **FT** *their* **(i)** or correct |
| (iii) | $g(1) = 5$ so $fg(1) = f(5)$ | M1 | evaluation using correct order of operations |
| | $5e^5 - 1 = 741$ | A1 | awrt 741 or $5e^5 - 1$ |
| (iv) | $g^2(x) = (x^2+4)^2 + 4$ | M1 | correct use of $g^2$ |
| | $x^4 + 8x^2 + 16 + 4 = 40$<br>$(x^2+4)^2 = 36$<br>or<br>$x^4 + 8x^2 - 20 = 0$<br>$(x^2+10)(x^2-2) = 0$ | M1 | **DepM1** for forming and solving a quadratic in $x^2$ |
| | $x = \pm\sqrt{2}$ only | A1 | |

**4**

$$f : x \mapsto (2x+3)^2 \quad \text{for } x > 0$$

**(a)** Find the range of f. [1]

**(b)** Explain why f has an inverse. [1]

**(c)** Find $f^{-1}$. [3]

**(d)** State the domain of $f^{-1}$. [1]

**(e)** Given that $g : x \mapsto \ln(x+4)$ for $x > 0$, find the exact solution of $fg(x) = 49$. [3]

www.smartexamresources.com

# MARKING SCHEME

| | | | |
|---|---|---|---|
| (a) | $f > 9$ | **B1** | Allow *y* but not *x* |
| (b) | It is a one-one function because of the restricted domain | **B1** | |
| (c) | $x = (2y+3)^2$ or equivalent | **M1** | For a correct attempt to find the inverse |
| | $y = \dfrac{\sqrt{x}-3}{2}$ | **M1** | For correct rearrangement |
| | $f^{-1} = \dfrac{\sqrt{x}-3}{2}$ | **A1** | Must have correct notation |
| (d) | $x > 9$ | **B1** | **FT** on *their* (a) |
| (e) | $f(\ln(x+4)) = 49$ | **M1** | For correct order |
| | $(2\ln(x+4)+3)^2 = 49$<br>$\ln(x+4) = 2$ | **M1** | For correct attempt to solve, dep on previous M mark, as far as $x =$ |
| | $x = e^2 - 4$ | **A1** | |

**5** Functions f and g are defined, for $x > 0$, by

$$f(x) = \ln x,$$
$$g(x) = 2x^2 + 3.$$

**(i)** Write down the range of f. [1]

**(ii)** Write down the range of g. [1]

**(iii)** Find the exact value of $f^{-1}g(4)$. [2]

**(iv)** Find $g^{-1}(x)$ and state its domain. [3]

www.smartexamresources.com

| | | | |
|---|---|---|---|
| (i) | $y \in \mathbb{R}$ oe | **B1** | Must have correct notation i.e. no use of $x$ |
| (ii) | $y > 3$ oe | **B1** | Must have correct notation i.e. no use of $x$ |
| (iii) | $f^{-1}(x) = e^x$ or $g(4) = 35$ | **B1** | First B1 may be implied by correct answer or by use of 35 |
| | $f^{-1}g(4) = e^{35}$ | **B1** | |
| (iv) | $\frac{y-3}{2} = x^2$ or $\frac{x-3}{2} = y^2$ | **M1** | valid attempt to obtain the inverse |
| | $g^{-1}(x) = \sqrt{\frac{x-3}{2}}$ | **A1** | correct form, must be $g^{-1}(x) =$ or $y =$ |
| | Domain $x > 3$ | **B1** | Must have correct notation |

www.smartexamresources.com

**6**

**(a)** It is given that

$$f : x \mapsto \sqrt{x} \quad \text{for } x \geqslant 0,$$
$$g : x \mapsto x + 5 \quad \text{for } x \geqslant 0.$$

Identify each of the following functions with one of $f^{-1}$, $g^{-1}$, $fg$, $gf$, $f^2$, $g^2$.

**(i)** $\sqrt{x+5}$ [1]

**(ii)** $x - 5$ [1]

**(iii)** $x^2$ [1]

**(iv)** $x + 10$ [1]

**(b)** It is given that $h(x) = a + \dfrac{b}{x^2}$ where $a$ and $b$ are constants.

**(i)** Why is $-2 \leqslant x \leqslant 2$ not a suitable domain for $h(x)$? [1]

**(ii)** Given that $h(1) = 4$ and $h'(1) = 16$, find the value of $a$ and of $b$. [2]

www.smartexamresources.com

# MARKING SCHEME

| | | | |
|---|---|---|---|
| a(i) | fg | **B1** | |
| a(ii) | $g^{-1}$ | **B1** | |
| a(iii) | $f^{-1}$ | **B1** | |
| a(iv) | $g^2$ | **B1** | |
| (b)(i) | Undefined at $x = 0$ oe | **B1** | |
| (b)(ii) | $4 = a + b$<br>$h'(x) = \frac{p}{x^3}$ and attempt at $h'(1)$ | **M1** | For attempt at $h(1)$ and differentiation to obtain $h'(1)$, must have the form $h'(x) = \frac{p}{x^3}$ oe |
| | $b = -8$<br>$a = 12$ | **A1** | For both |

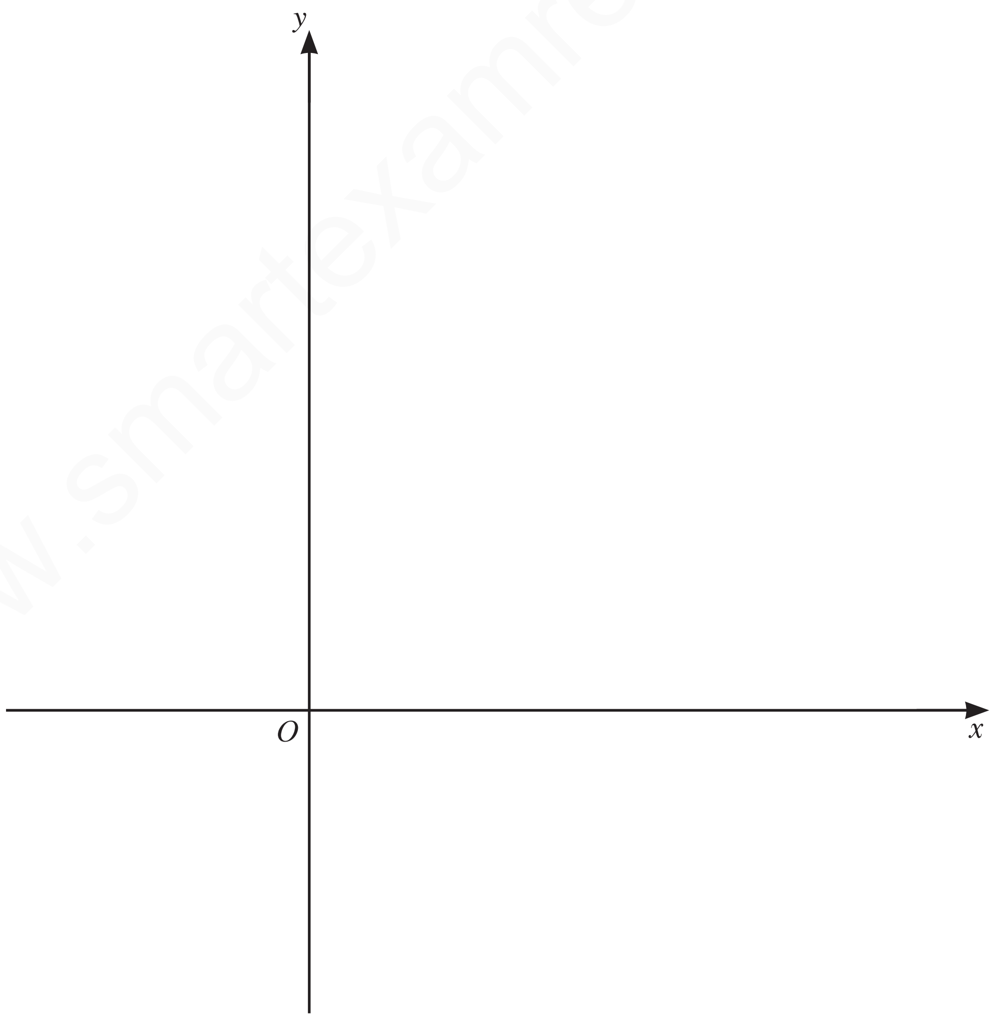
www.smartexamresources.com

**7** It is given that $f(x) = 5\ln(2x+3)$ for $x > -\frac{3}{2}$.

**(a)** Write down the range of f. [1]

**(b)** Find $f^{-1}$ and state its domain. [3]

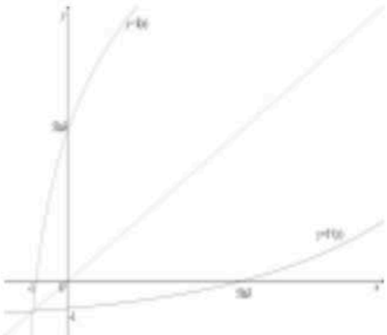
**(c)** On the axes below, sketch the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. Label each curve and state the intercepts on the coordinate axes.




[5]

www.smartexamresources.com

# MARKING SCHEME

| | | | |
|---|---|---|---|
| (a) | $f \in \mathbb{R}$ | **B1** | Allow *y* but not *x* |
| (b) | $x = 5\ln(2y+3)$<br>$e^{\frac{x}{5}} = 2y+3$ | **M1** | For a complete attempt to obtain inverse |
| | $f^{-1}(x) = \dfrac{e^{\frac{x}{5}} - 3}{2}$ | **A1** | Must be using correct notation |
| | Domain $x \in \mathbb{R}$ | **B1** | **FT** on *their* **(a)**. Must be using correct notation |
| (c) |  | **5** | **B1** for shape of $y = f(x)$<br>**B1** for shape of $y = f^{-1}(x)$<br>**B1** for $5\ln 3$ or 5.5 and $-1$ on both axes for $y = f(x)$<br>**B1** for $5\ln 3$ or 5.5 and $-1$ on both axes for $y = f^{-1}(x)$<br>**B1** All correct, with apparent symmetry which may be implied be previous 2 B marks or by inclusion of $y = x$, and implied asymptotes, may have one or two points of intersection |