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0607/06

**May/June 2013**

**1 hour 30 minutes**

Additional Materials: Graphics Calculator

DO **NOT** WRITE IN ANY BARCODES.

The total number of marks for this paper is 40.



**[Turn over**

Answer **both** parts A and B.

For  
Examiner's  
Use

# **A INVESTIGATION DIAGONALS OF RECTANGLES (20 marks)**

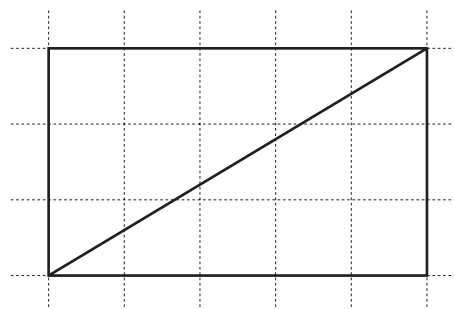
You are advised to spend 45 minutes on part A.

Rectangles are drawn on a grid.

The sides of each rectangle lie on gridlines and the length is greater than or equal to the width.

This investigation looks for a method for calculating the number of small squares through which a diagonal passes.

- 1 The diagram shows a rectangle with length 5 and width 3.  
The diagonal crosses 4 vertical gridlines **inside** the rectangle.



Write down

- (a) the number of horizontal gridlines that the diagonal crosses inside the rectangle,

.....

- (b) the total number of gridlines that the diagonal crosses inside the rectangle.

.....

- 2 A rectangle has length  $x$  and width  $y$ .  
 $x$  and  $y$  do not have a common factor.

- (a) Write down an expression for

- (i) the number of vertical gridlines that a diagonal crosses inside the rectangle, in terms of  $x$ ,

.....

- (ii) the number of horizontal gridlines that a diagonal crosses inside the rectangle, in terms of  $y$ ,

.....

- (iii) the total number of gridlines,  $N$ , which a diagonal crosses inside the rectangle, in terms of  $x$  and  $y$ . Write your answer in its simplest form.

$N =$  .....

- (b)  $S$  is the number of squares through which the diagonal passes.  
For example, the diagonal in **question 1** passes through 7 squares.

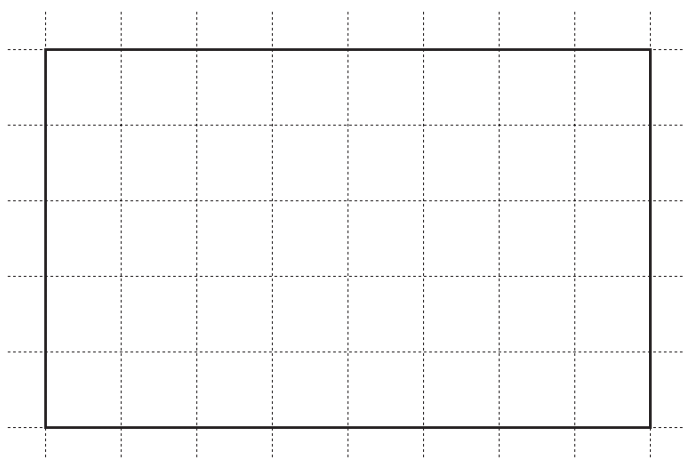
(i) Write  $S$  in terms of  $N$ .

$$S = \dots\dots\dots$$

(ii) Write  $S$  in terms of  $x$  and  $y$ .

$$S = \dots\dots\dots$$

- (c) Show that your formula for  $S$  in **part (b)(ii)** gives the correct value for an 8 by 5 rectangle.  
Use the grid to show clearly how many squares the diagonal passes through.



.....

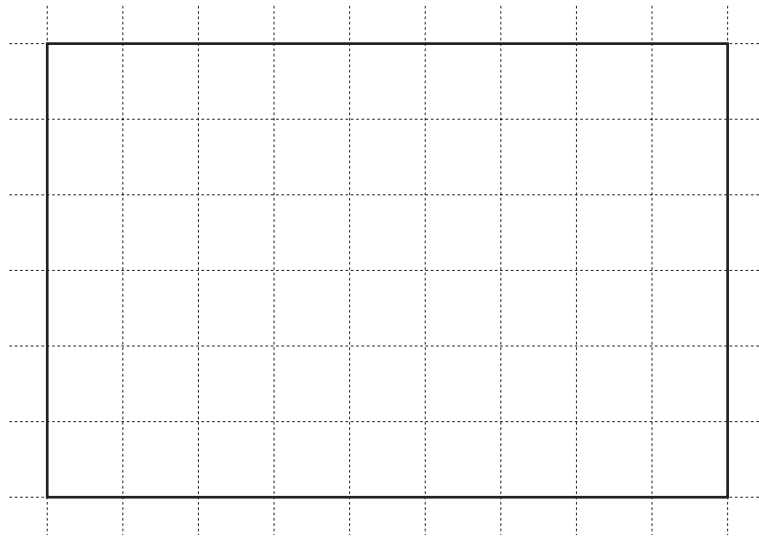
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- 3 In **question 2**,  $x$  and  $y$  did not have a common factor.  
In this question,  $x$  and  $y$  do have a common factor.

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Use*

- (a) (i) Show clearly that your formula for  $S$  does not give the correct value for a 9 by 6 rectangle.



- (ii) 9 and 6 have a common factor of 3.  
Show how you use the value of  $S$  for a 3 by 2 rectangle to calculate  $S$  for a 9 by 6 rectangle.

(b) Use your method in **part (a)(ii)** to find  $S$  for each of these rectangles.

(i) 93 by 90

.....

(ii) 60 by 35

.....

4 The diagonal of a rectangle passes through 6 squares.

Use **question 2** and **question 3** to find the length and the width of each possible rectangle.

*For  
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Use*

**B MODELLING****DRILLING A TUNNEL (20 marks)**

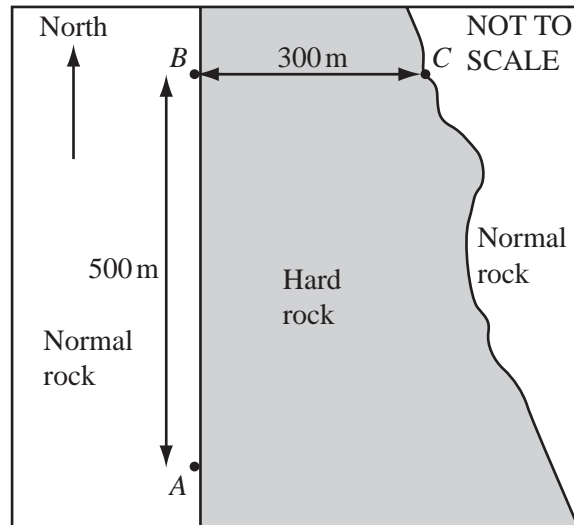
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You are advised to spend 45 minutes on part **B**.

On the plan,  $A$  is south of  $B$  and  $C$  is east of  $B$ .

$AB = 500$  metres and  $BC = 300$  metres.

Engineers want to drill a tunnel from  $A$  to  $C$ .  
The tunnel has one or more straight sections.



- 1 Calculate the length of the shortest possible tunnel from  $A$  to  $C$ .  
Give your answer correct to the nearest metre.

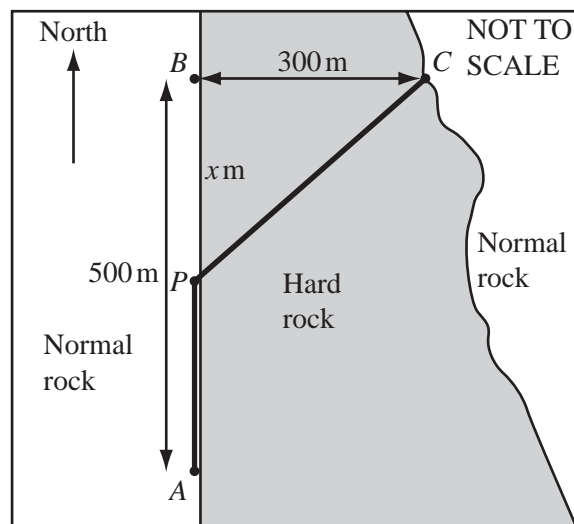
..... m

- 2 Write down the length of the tunnel if the engineers drill through as little hard rock as possible.

..... m

- 3  $P$  is a point which is  $x$  metres south of  $B$ .

The engineers decide to drill from  $A$  to  $P$  to  $C$ .



Through normal rock, from  $A$  to  $P$ , the drill moves forward at 2 metres per hour.  
Through the hard rock, from  $P$  to  $C$ , the drill moves forward at 1 metre per hour.

- (a) Explain why the time in hours,  $T$ , that it takes to drill the tunnel, can be modelled by this equation.

$$T = \frac{500 - x}{2} + \sqrt{90000 + x^2}$$

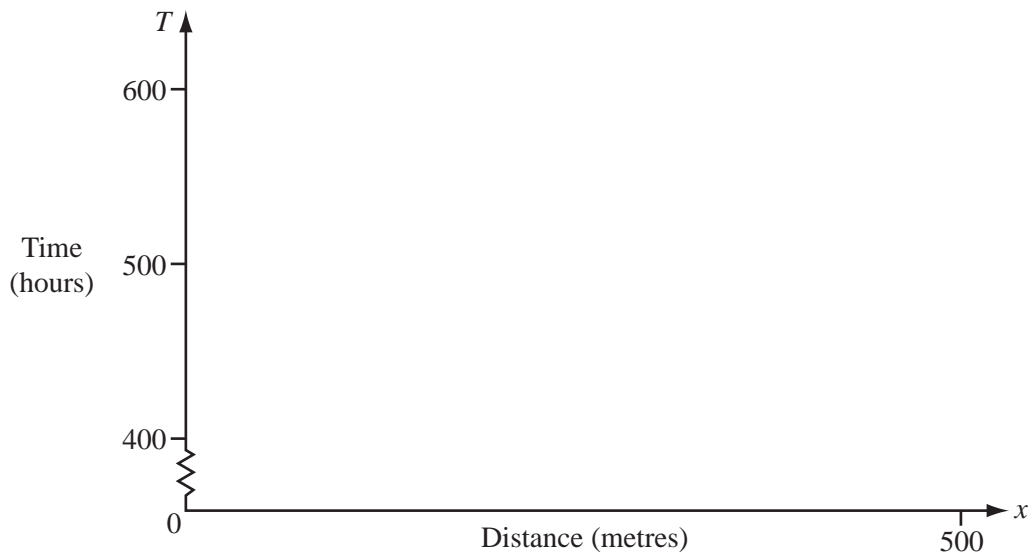
For  
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Use

- (b) All the measurements are accurate.  
Write down a practical reason why the time given by the model may be different from the actual time.

.....

.....

- (c) On the diagram, sketch the graph of  $T$  against  $x$ .



- (d) (i) Find, to the nearest metre, the position of  $P$  which gives the minimum time to drill the tunnel.

..... metres from  $B$

- (ii) Find this minimum time correct to the nearest 10 hours.

..... hours

- 4 To drill through normal rock costs 2 thousand dollars per hour.  
To drill through the hard rock costs 3 thousand dollars per hour.

- (a) The total cost of drilling the tunnel is  $n$  thousand dollars.  
Write down a model for  $n$  in terms of  $x$ .

$n =$  .....

- (b) (i) Find, to the nearest metre, the position of  $P$  which gives the minimum cost.

..... metres from  $B$

- (ii) Write, in full, this minimum cost to the nearest ten thousand dollars.

\$ .....

- 5 The model for the time taken to drill the tunnel is  $T = \frac{500-x}{2} + \sqrt{90000 + x^2}$ .

- (a) The position of  $B$  and  $C$  are fixed.  
Investigate the position of  $P$  which gives the minimum time when  $A$  is more than 500 m south of  $B$ .

- (b) If  $AB = d$  metres explain, using **part (a)**, why the minimum time in hours is

$$T = \frac{d}{2} + k, \text{ where } k = 260 \text{ correct to 3 significant figures.}$$