FUNCTIONS-SET-2-QP-MS

1 Answer only **one** of the following two alternatives.

EITHER

A function f is such that $f(x) = \ln(5x - 10)$, for $x > 2$.

(i) State the range of f. [1]

(ii) Find $f^{-1}(x)$. [3]

(iii) State the range of f^{-1}. [1]

(iv) Solve $f(x) = 0$. [2]

A function g is such that $g(x) = 2x - \ln 2$, for $x \in \mathbb{R}$.

(v) Solve $gf(x) = f(x^2)$. [5]

OR

A function f is such that $f(x) = 4e^{-x} + 2$, for $x \in \mathbb{R}$.

(i) State the range of f. [1]

(ii) Solve $f(x) = 26$. [2]

(iii) Find $f^{-1}(x)$. [3]

(iv) State the domain of f^{-1}. [1]

A function g is such that $g(x) = 2e^{x} - 4$, for $x \in \mathbb{R}$.

(v) Using the substitution $t = e^{x}$ or otherwise, solve $g(x) = f(x)$. [5]

Start your answer to Question 12 here.

Indicate which question you are answering.

EITHER	
OR	

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Continue your answer to Question 12 here.

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MARKING SCHEME:

EITHER		
(i) $\mathbb{R}$ or equivalent	B1	
(ii) $e^y = 5x - 10$, $\dfrac{e^y + 10}{5} = x$	M1	M1 rearrangement to x in terms of y
$f^{-1}(x) = \dfrac{e^x + 10}{5}$	DM1 A1	DM1 for interchange of x and y A1 for correct form
(iii) $f^{-1}(x) > 2$ or $y > 2$	B1	
(iv) $1 = 5x - 10$ $x = 2.2$	B1 B1	
(v) $g(\ln(5x-10)) = \ln(5x^2 - 10)$ $2\ln(5x-10) - \ln 2 = \ln(5x^2 - 10)$ $25x^2 - 100x - 100 = 10x^2 - 20$ $3x^2 - 20x + 24 = 0$, leading to $x = 5.10$ only	M1 M1 A1 M1 A1 [12]	M1 for correct order gf M1 for dealing with x^2 correctly A1 correct quadratic– allow unsimplified M1 for correct attempt at solution of a 3 term quadratic A1 for valid solution only

OR		
(i) $f(x) > 2$	B1	
(ii) $26 = 4e^{-x} + 2$	B1	
$6 = e^{-x}$ so $x = -\ln 6$, $\ln\dfrac{1}{6}$ or -1.79	B1	
(iii) $\dfrac{(y-2)}{4} = e^{-x}$, $\ln\dfrac{(y-2)}{4} = -x$	M1	M1 rearrangement to x in terms of y
$f^{-1}(x) = \ln\dfrac{4}{x-2}$ or $-\ln\dfrac{x-2}{4}$	M1 A1	M1 for interchange of x and y A1 for correct form
(iv) $f^{-1}(x)$ or $y > 2$	B1	
(v) $2e^x - 4 = 4e^{-x} + 2$ $(2t - 4 = 4t^{-1} + 2)$	M1 A1	M1 for attempt to deal with t^{-1} or e^{-x} A1 for correct quadratic equation
$e^{2x} - 3e^x - 2 = 0$ $(t^2 - 3t - 2 = 0)$ $e^x = 3.56$ so $x = 1.27$	M1 M1 A1 [12]	M1 for solution of quadratic M1 for correct attempt to obtain x A1 for 1 solution only

2 A function g is such that $g(x) = \frac{1}{2x - 1}$ for $1 \leqslant x \leqslant 3$.

(i) Find the range of g. [1]

(ii) Find $g^{-1}(x)$. [2]

(iii) Write down the domain of $g^{-1}(x)$. [1]

(iv) Solve $g^2(x) = 3$. [3]

MARKING SCHEME:

(i) $0.2 \leqslant y \leqslant 1$	B1 [1]	Must be using correct notation
(ii) $g^{-1}(x) = \frac{1+x}{2x}$	M1 A1 [2]	M1 for a valid method to find inverse A1 must have correct notation
(iii) $0.2 \leqslant x \leqslant 1$	√B1 [1]	Follow through on their **(i)**
(iv) $g^2 = \frac{1}{2\left(\frac{1}{2x-1}\right)-1} = 3$ $\frac{2x-1}{3-2x} = 3$ leading to $x = 1.25$	M1 DM1 A1 [3]	M1 for correct attempt to find g^2 DM1 for equating to 3 and attempt to solve.

3 **(a)** A function f is such that $f(x) = 3x^2 - 1$ for $-10 \leqslant x \leqslant 8$.

(i) Find the range of f. [3]

(ii) Write down a suitable domain for f for which f^{-1} exists. [1]

(b) Functions g and h are defined by

$$g(x) = 4e^x - 2 \text{ for } x \in \mathbb{R},$$

$$h(x) = \ln 5x \text{ for } x > 0.$$

(i) Find $g^{-1}(x)$. [2]

(ii) Solve $gh(x) = 18$. [3]

(a) **(i)** f(–10) = 299, f(8) = 191 Min point at (0, –1) or when $y = -1$ $\therefore$ range $-1 \le y \le 299$	M1 B1 A1 [3]	M1 for substitution of either $x = -10$ or $x = 8$, may be seen on diagram B1 May be implied from final answer, may be seen on diagram Must have $\le$ for A1, do not allow x
(ii) $x \ge 0$ or equivalent	B1 [1]	Allow any domain which will make f a one-one function Assume upper and lower bound when necessary.
(b) **(i)** $g^{-1}(x) = \ln\left(\frac{x+2}{4}\right)$ or $\dfrac{\lg\left(\frac{x+2}{4}\right)}{\lg e}$	M1 A1 [2]	M1 for complete method to find the form inverse function, must involve ln or lg if appropriate. May still be in terms of y. A1 must be in terms of x
(ii) $gh(x) = g(\ln 5x)$ $= 4e^{\ln 5x} - 2$ $20x - 2 = 18$, $x = 1$	M1 A1 A1 [3]	M1 for correct order A1 for correct expression $4e^{\ln 5x} - 2$ A1 for correct solution from correct working
Or $h(x) = g^{-1}(18)$ $\ln 5x = \ln 5$ leading to $x = 1$	M1 A1 A1	M1 for correct order A1 for correct equation A1 for correct solution **from correct working**

4 The function f is defined by

$$f(x) = (2x + 1)^2 - 3 \quad \text{for } x \geqslant -\frac{1}{2}.$$

Find

(i) the range of f, [1]

(ii) an expression for $f^{-1}(x)$. [3]

The function g is defined by

$$g(x) = \frac{3}{1 + x} \quad \text{for } x > -1.$$

(iii) Find the value of x for which $fg(x) = 13$. [4]

MARKING SCHEME:

(i) $f \geqslant -3$	B1 [1]	
(ii) $f^{-1} = \dfrac{\sqrt{x+3}-1}{2}$	M1 M1 A1 [3]	M1 for correct order of operations M1 for 'interchange' of x and y
(iii) $\left(2\left(\dfrac{3}{1+x}\right)+1\right)^2 - 3 = 13$	M1	M1 for correct order
$\left(\dfrac{7+x}{1+x}\right)^2 = 16$ $x = 1$	A1 M1 B1 [4]	A1 for correct simplification M1 for solution B1 for **one** solution only

5 The function f is given by f: $x \mapsto 5 - 3e^{\frac{1}{2}x}$, $x \in \mathbb{R}$.

(i) State the range of f. [1]

(ii) Solve the equation $f(x) = 0$, giving your answer correct to two decimal places. [2]

(iii) Sketch the graph of $y = f(x)$, showing on your diagram the coordinates of the points of intersection with the axes. [2]

(iv) Find an expression for f^{-1} in terms of x. [3]

MARKING SCHEME:

$f(x) = 5-3e^{\frac{1}{2}x}$		
(i) Range is <5	B1	Allow $\le$ or $<$
(ii) $5-3e^{\frac{1}{2}x} = 0 \rightarrow e^{\frac{1}{2}x} = 5/3$ Logs or calculator $\rightarrow x = 1.02$	M1A1	Normally 2,0 but if working shown, can get M1 if appropriate
(iii) (1.02, 0) and (0, 2)	B1 B1√	Shape in 1st quadrant. Both shown or implied by statement.
(iv) $e^{\frac{1}{2}x} = (5-y)\div 3$ $x/2 = \ln[(5-y)/3]$ $f^{-1}(x) = 2\ln[(5-x)/3]$	M1 M1 A1	Reasonable attempt $e^{\frac{1}{2}x}$ as the subject. Using logs. All ok, including x, y interchanged.
	[8]	

6 A function f is defined by $f : x \mapsto |2x - 3| - 4$, for $-2 \leqslant x \leqslant 3$.

(i) Sketch the graph of $y = f(x)$. [2]

(ii) State the range of f. [2]

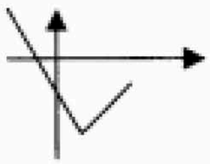
(iii) Solve the equation $f(x) = -2$. [3]

A function g is defined by $g : x \mapsto |2x - 3| - 4$, for $-2 \leqslant x \leqslant k$.

(iv) State the largest value of k for which g has an inverse. [1]

(v) Given that g has an inverse, express g in the form $g : x \mapsto ax + b$, where a and b are constants. [2]

MARKING SCHEME:

$f : x \rightarrow \|2x-3\|-4 \quad -2 \leq x \leq 3$ (i) 	B2,1 [2]	Must be "V" shaped to get any marks. Must cross −ve x and −ve y axes. Endpoint −ve y. Start point + ve y.
(ii) Range of f −4 to 3	B1 B1 [2]	Independent of graph. −4 on own ok. 3 on its own.
(iii) $2x-3=2 \rightarrow x = 2\frac{1}{2}$ or 2.5 $2x-3=-2 \rightarrow x = \frac{1}{2}$ or 0.5	B1 M1A1 [3]	Co – answer only Correct method of other solution. co
(iv) Largest value is x value at "V" = $1\frac{1}{2}$	B1√ [1]	From his graph – or any other method
(v) Equation of left hand part of "V". $m = -2 \rightarrow -2x-1$.	M1 A1 [2]	Realises that one line only is needed + correct method ($y=mx+c$ etc). Or $-(2x-3)-4=-2x-1$ Doesn't need a or b implicitly mentioned