FUNCTIONS-SET-6-QP-MS

1 **(a)** $f(x) = 4\ln(2x-1)$

(i) Write down the largest possible domain for the function f. [1]

(ii) Find $f^{-1}(x)$ and its domain. [3]

(b)

$g(x) = x+5 \quad \text{for } x \in \mathbb{R}$

$h(x) = \sqrt{2x-3} \quad \text{for } x \geqslant \frac{3}{2}$

Solve $gh(x) = 7$. [3]

www.smartexamresources.com

MARKING SCHEME

(a)(i)	$x > \frac{1}{2}$	B1	Must be using x
(a)(ii)	$x = 4\ln(2y-1)$ $e^{\frac{x}{4}} = 2y-1$ $y = \frac{1}{2}\left(1+e^{\frac{x}{4}}\right)$	M1	For full method for inverse using correct order of operations
	$f^{-1}(x) = \frac{1}{2}\left(1+e^{\frac{x}{4}}\right)$ or $f^{-1}(x) = \frac{1}{2}\left(1+\sqrt[4]{e^x}\right)$	A1	Must be using correct notation
	$x \in \mathbb{R}$	B1	
(b)	$\sqrt{2x-3}+5=7$	M1	For correct order
	$x = \frac{2^2+3}{2}$	M1	Dep on previous M mark, for obtaining x by simplifying and solving using correct order of operations, including squaring
	$x = \frac{7}{2}$ or 3.5	A1	

2 $\quad \mathrm{f} : x \mapsto \mathrm{e}^{3x} \quad \text{for } x \in \mathbb{R}$

$\mathrm{g} : x \mapsto 2x^2 + 1 \quad \text{for } x \geqslant 0$

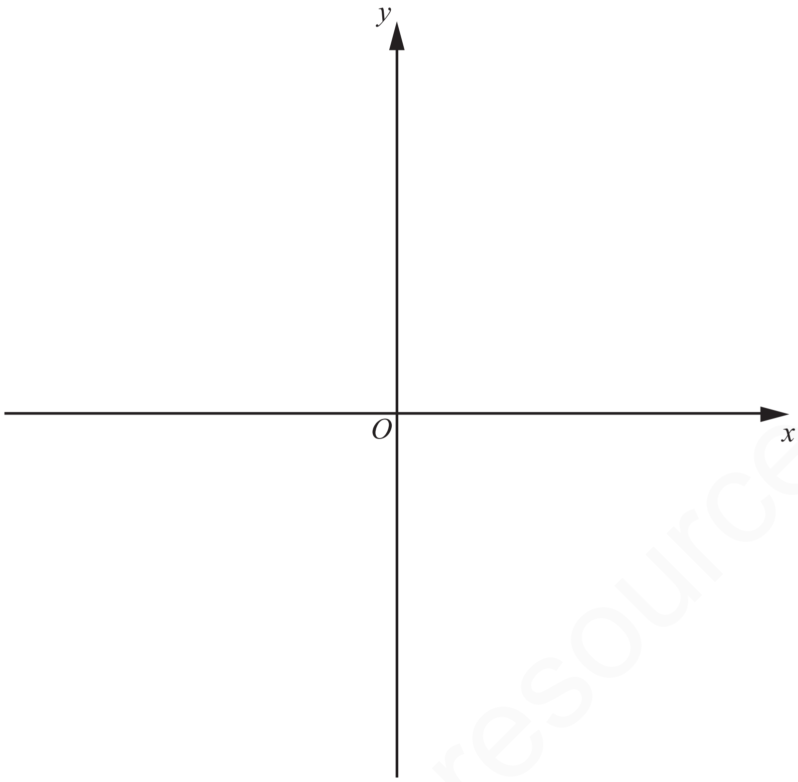
(i) Write down the range of g. [1]

(ii) Show that $\mathrm{f}^{-1}\mathrm{g}(\sqrt{62}) = \ln 5$. [3]

(iii) Solve $\mathrm{f}'(x) = 6\mathrm{g}''(x)$, giving your answer in the form $\ln a$, where a is an integer. [3]

www.smartexamresources.com

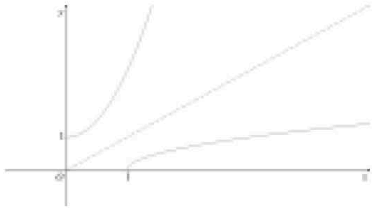
(iv) On the axes below, sketch the graph of $y = \mathrm{g}$ and the graph of $y = \mathrm{g}^{-1}$, showing the points where the graphs meet the coordinate axes.



[3]

www.smartexamresources.com

MARKING SCHEME

(i)	$g \geqslant 1$	**B1**	Must be using correct notation
(ii)	$g\left(\sqrt{62}\right) = 125$	**B1**	
	$f^{-1}(x) = \frac{1}{3}\ln x$	**B1**	
	$\frac{1}{3}\ln 125 = \ln 5$	**B1**	For correct order and manipulation to obtain the given answer, need to see $\frac{1}{3}\ln 125$
(iii)	$3e^{3x} = 24$	**M1**	For dealing with derivatives correctly
	$x = \frac{1}{3}\ln 8$	**A1**	
	$x = \ln 2$	**A1**	
(iv)		**B3**	**B1** for correct g with intercept **B1** for $y = x$ and/or implication of symmetry **B1** for correct g^{-1} with intercept

3

$$f(x) = 3 + e^{x} \quad \text{for } x \in \mathbb{R}$$

$$g(x) = 9x - 5 \quad \text{for } x \in \mathbb{R}$$

(a) Find the range of f and of g. [2]

(b) Find the exact solution of $f^{-1}(x) = g'(x)$. [3]

(c) Find the solution of $g^{2}(x) = 112$. [2]

www.smartexamresources.com

MARKING SCHEME

(a)	$f > 3$	**B1**	Allow y but not x
	$g \in \mathbb{R}$	**B1**	Allow y but not x
(b)	$\ln(x-3)$	**B1**	
	$\ln(x-3) = 9$ $x - 3 = e^9$	**M1**	For attempt to equate to 9 and solve, must get rid of ln
	$x = e^9 + 3$	**A1**	
(c)	$9(9x-5) - 5 = 112$	**M1**	For correct order of operation
	$x = 2$	**A1**	

www.smartexamresources.com

4 Functions f and g are defined, for $x > 0$, by

$$\mathrm{f}(x) = \ln x,$$
$$\mathrm{g}(x) = 2x^2 + 3.$$

(i) Write down the range of f. [1]

(ii) Write down the range of g. [1]

(iii) Find the exact value of $\mathrm{f}^{-1}\mathrm{g}(4)$. [2]

(iv) Find $\mathrm{g}^{-1}(x)$ and state its domain. [3]

www.smartexamresources.com

MARKING SCHEME

(i)	$y \in \mathbb{R}$ oe	**B1**	Must have correct notation i.e. no use of x
(ii)	$y > 3$ oe	**B1**	Must have correct notation i.e. no use of x
(iii)	$f^{-1}(x) = e^x$ or $g(4) = 35$	**B1**	First B1 may be implied by correct answer or by use of 35
	$f^{-1}g(4) = e^{35}$	**B1**	
(iv)	$\frac{y-3}{2} = x^2$ or $\frac{x-3}{2} = y^2$	**M1**	valid attempt to obtain the inverse
	$g^{-1}(x) = \sqrt{\frac{x-3}{2}}$	**A1**	correct form, must be $g^{-1}(x) =$ or $y =$
	Domain $x > 3$	**B1**	Must have correct notation

5 **(a)** f( ) = $-$ x $\leqslant x \leqslant \frac{\pi}{2}$. co3 2 s for 0

(i) Write down the range of f. [2]

(ii) Find the exact value of $f^{-1}(2.5)$. [3]

www.smartexamresources.com

(b) $g(x) = 3 - x^2$ for $x \in \mathbb{R}$.

Find the exact solutions of $g^2(x) = -6$. [4]

www.smartexamresources.com

(a)(i)		**B1**	For critical values
	$2 \leqslant f \leqslant 4$	**B1**	**Dep** For correct inequality and notation
(a)(ii)	$x = 3\cos y$ $\cos 2y = 0.5$	**M1**	For attempt to find f^{-1} and equate to 0.5
	$2y = \frac{\pi}{3}$	**M1**	**Dep** For correct attempt to solve, dealing with the double angle
	$y = \frac{\pi}{6}$	**A1**	
(b)	$g^2(x) = g(3 - x^2)$ $= 3 - (3 - x^2)^2$	**M1**	For correct attempt at g^2, allow unsimplified
	$-6 + 6x^2 - x^4 = -6$ $6x^2 - x^4 = 0$	**M1**	**Dep** for equating to –6 and attempt to solve to obtain a non-zero root
	$x = 0$	**B1**	
	$x = \pm\sqrt{6}$	**A1**	

6

$$f(x) = 5 + \sin\frac{x}{4} \quad \text{for} \quad 0 \leqslant x \leqslant 2\pi \text{ radians}$$

$$g(x) = x - \frac{\pi}{3} \quad \text{for} \quad x \in \mathbb{R}$$

(i) Write down the range of $f(x)$. [2]

(ii) Find $f^{-1}(x)$ and write down its range. [3]

(iii) Solve $2fg(x) = 11$. [4]

www.smartexamresources.com

(i)	$5 \leqslant f(x) \leqslant 6$ or $[5,6]$ oe	B2	**B1** for $5 \leqslant f(x) \leqslant p\ (p>5)$ or for $q \leqslant f(x) \leqslant 6\ (q<6)$
(ii)	$x = \sin\frac{y}{4} + 5$	M1	complete valid attempt to obtain the inverse with operations in correct order.
	$y = 4\sin^{-1}(x-5)$	A1	
	Range $0 \leqslant y \leqslant 2\pi$	B1	
(iii)	$2\left(\sin\frac{\left(x-\frac{\pi}{3}\right)}{4}+5\right)$ $(=11)$	B1	for $\sin\frac{\left(x-\frac{\pi}{3}\right)}{4}+5$
	$\sin\frac{\left(x-\frac{\pi}{3}\right)}{4} = \frac{1}{2}$	M1	for $\sin\frac{\left(x-\frac{\pi}{3}\right)}{4} = k$
	$x = 4\sin^{-1}\left(\frac{1}{2}\right) + \frac{\pi}{3}$ oe	M1	**Dep** for use of $\sin^{-1}$ and correct order of operations to obtain x. Allow one +/– or ×/÷ sign error
	$x = \pi$ or 3.14	A1	$x = \pi$ and no other solutions in range

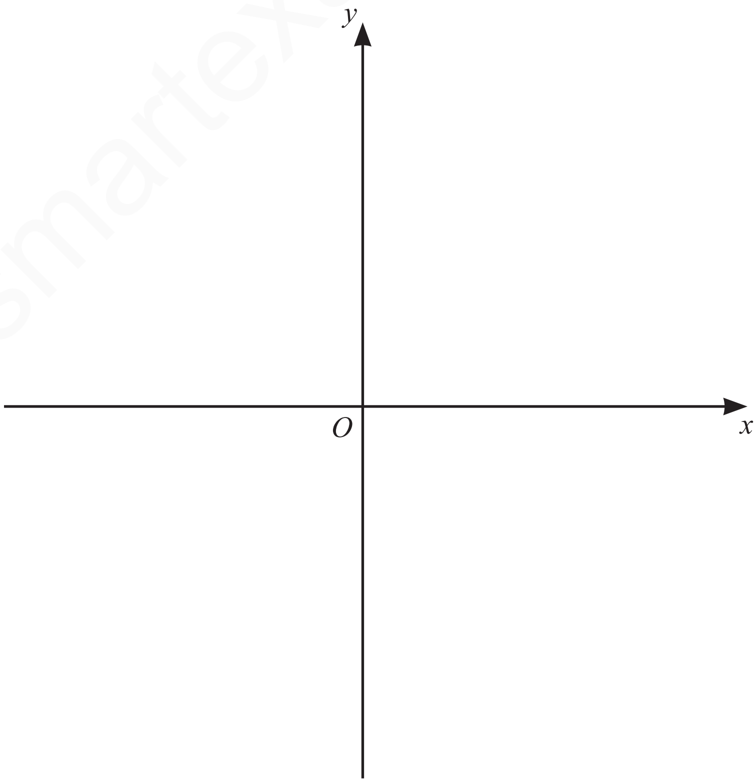
7

$$f(x) = 3e^{2x} + 1 \quad \text{for } x \in \mathbb{R}$$
$$g(x) = x + 1 \quad \text{for } x \in \mathbb{R}$$

(i) Write down the range of f and of g. [2]

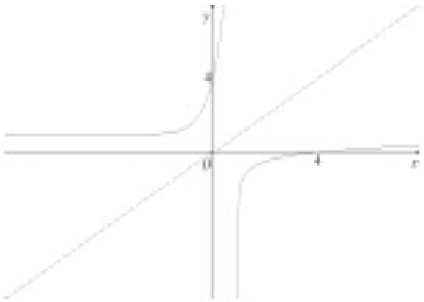
(ii) Evaluate $fg^2(0)$. [2]

(iii) On the axes below, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, stating the coordinates of the points where the graphs meet the coordinate axes. [3]



www.smartexamresources.com

MARKING SCHEME

(i)	$f > 1$	**B1**	Must be using correct notation
	$g \in \mathbb{R}$	**B1**	Must be using correct notation
(ii)	$g(0) = 1,\ g(1) = 2$ and attempt at $f(2)$	**M1**	For attempt at g^2 and correct order
	$f(2) = 164.8$ awrt 165	**A1**	
(iii)		**B3**	**B1** for correct f and $(0, 4)$, must be in first and second quadrant **B1** for correct f^{-1} and $(4, 0)$, must be in first and fourth quadrant **B1** for $y = x$ and/or symmetry implied, by 'matching intercepts'. No intersection.

www.smartexamresources.com