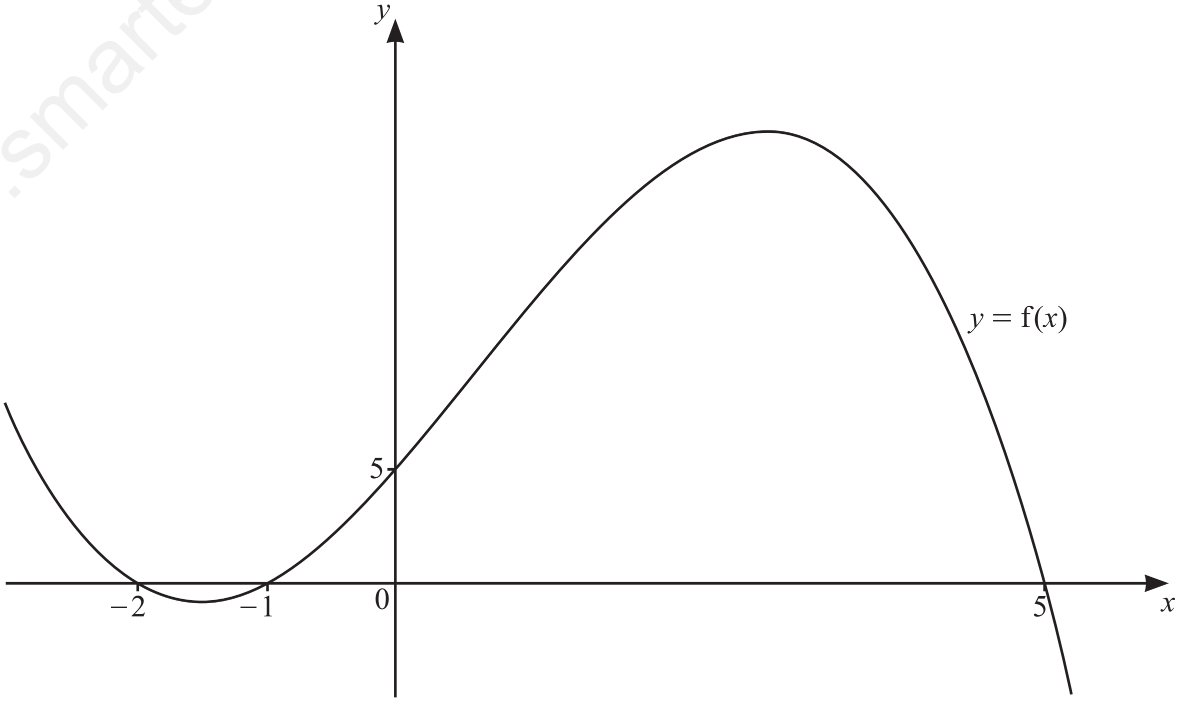
TOPIC: FUNCTIONS-SET-8

1 The diagram shows the graph of a cubic curve $y = \mathrm{f}(x)$.



(a) Find an expression for $\mathrm{f}(x)$. [2]

(b) Solve $\mathrm{f}(x) \leqslant 0$. [2]

MARK SCHEME:

(a)	$y=-\frac{1}{2}(x+2)(x+1)(x-5)$	**B1**	For $-\frac{1}{2}$
		B1	For $(x+2)(x+1)(x-5)$
(b)	$-2 \leqslant x \leqslant -1$	**B1**	
	$x \geqslant 5$	**B1**	

2 $$f : x \mapsto (2x+3)^2 \quad \text{for } x > 0$$

(a) Find the range of f. [1]

(b) Explain why f has an inverse. [1]

(c) Find f^{-1}. [3]

(d) State the domain of f^{-1}. [1]

(e) Given that $g : x \mapsto \ln(x+4)$ for $x > 0$, find the exact solution of $fg(x) = 49$. [3]

MARK SCHEME:

(a)	$f > 9$	**B1**	Allow y but not x
(b)	It is a one-one function because of the restricted domain	**B1**	
(c)	$x = (2y+3)^2$ or equivalent	**M1**	For a correct attempt to find the inverse
	$y = \frac{\sqrt{x}-3}{2}$	**M1**	For correct rearrangement
	$f^{-1} = \frac{\sqrt{x}-3}{2}$	**A1**	Must have correct notation
(d)	$x > 9$	**B1**	**FT** on *their* (a)
(e)	$f(\ln(x+4)) = 49$	**M1**	For correct order
	$(2\ln(x+4)+3)^2 = 49$ $\ln(x+4) = 2$	**M1**	For correct attempt to solve, dep on previous M mark, as far as $x =$
	$x = e^2 - 4$	**A1**	

3

$$\mathrm{f}(x) = 3 + \mathrm{e}^x \quad \text{for } x \in \mathbb{R}$$

$$\mathrm{g}(x) = 9x - 5 \quad \text{for } x \in \mathbb{R}$$

(a) Find the range of f and of g. [2]

(b) Find the exact solution of $\mathrm{f}^{-1}(x) = \mathrm{g}'(x)$. [3]

(c) Find the solution of $\mathrm{g}^2(x) = 112$. [2]

MARK SCHEME:

(a)	$f > 3$	**B1**	Allow y but not x
	$g \in \mathbb{R}$	**B1**	Allow y but not x
(b)	$\ln(x-3)$	**B1**	
	$\ln(x-3) = 9$ $x - 3 = e^9$	**M1**	For attempt to equate to 9 and solve, must get rid of ln
	$x = e^9 + 3$	**A1**	
(c)	$9(9x-5) - 5 = 112$	**M1**	For correct order of operation
	$x = 2$	**A1**	

4 The functions f and g are defined as follows.

$$f(x) = x^2 + 4x \quad \text{for } x \in \mathbb{R}$$

$$g(x) = 1 + e^{2x} \quad \text{for } x \in \mathbb{R}$$

(a) Find the range of f. [2]

(b) Write down the range of g. [1]

(c) Find the exact solution of the equation $fg(x) = 21$, giving your answer as a single logarithm. [4]

MARK SCHEME:

(a)	$f \geqslant -4$	2	**M1** for a valid method to find the least value of $x^2 + 4x$ **A1** for $f \geqslant -4$, $y \geqslant -4$ or $f(x) \geqslant -4$
(b)	$g > 1$	**B1**	Allow $y > 1$ or $g(x) > 1$
(c)	$\left(1+e^{2x}\right)^2 + 4\left(1+e^{2x}\right)[=21]$	**M1**	
	$e^{4x} + 6e^{2x} - 16 = 0$ $\left(e^{2x}+8\right)\left(e^{2x}-2\right) = 0$	**M1**	**Dep** for quadratic in terms of e^{2x} and attempt to solve to obtain $e^{2x} = k$
	$e^{2x} = 2$ $x = \frac{1}{2}\ln k$	**M1**	**Dep** on both previous M marks, for attempt to solve $e^{2x} = k$
	$x = \ln\sqrt{2}$ or $\ln 2^{\frac{1}{2}}$	**A1**	

5 A function f is such that $f(x) = \ln(2x+1)$, for $x > -\frac{1}{2}$.

(a) Write down the range of f. [1]

A function g is such that $g(x) = 5x - 7$, for $x \in \mathbb{R}$.

(b) Find the exact solution of the equation $gf(x) = 13$. [3]

(c) Find the solution of the equation $f'(x) = g^{-1}(x)$. [6]

MARK SCHEME:

(a)	$f(x) \in \mathbb{R}$ oe	**B1**	Must be using correct notation, allow $y \in$
(b)	$5(\ln(3x+1)) - 7 = 13$	**M1**	For correct order
	$x = \dfrac{e^4 - 1}{2}$	**2**	**M1** for a correct attempt to solve to get $x =$, allow one sign error **Dep** on previous M mark **A1** all correct must be exact
(c)	$(f'(x) =) \dfrac{2}{2x+1}$	**2**	**M1** for $\dfrac{a}{2x+1}$ **A1** all correct
	$(g^{-1}(x) =) \dfrac{x+7}{5}$	**B1**	soi
	$2x^2 + 15x - 3 = 0$	**M1**	for equating and forming a 3-term quadratic equation = 0
	$x = 0.195, -7.69$	**M1**	For solution of *their* 3-term quadratic
	$x = 0.195$	**A1**	For discounting negative root.